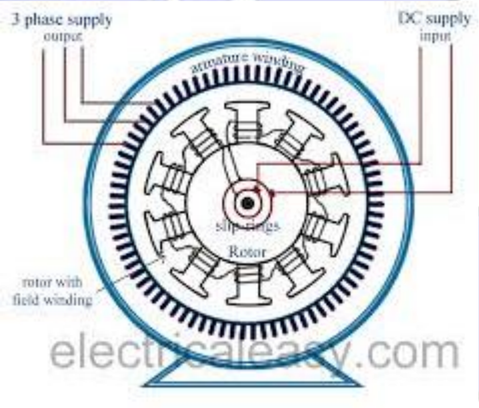
AC Machines II

Synchronous Machines

1-1 Introduction:

A synchronous machine is an A.C. machine in which the rotor moves at a speed, which bears a constant relationship to the frequency of the current in the armature winding. As a motor, the shaft speed must remain constant irrespective of the load, provided that the supply frequency remains constant. As a generator, the speed must remain constant if the frequency of the output is not to vary. The field of a synchronous machine is a steady one. In very small machines this field may be produced by permanent magnets, but in most cases the field is excited by a direct current obtained from an auxiliary generator, which is mechanically coupled to the shaft of the main machine.



1-2 Types of Synchronous Machines:

The armature or main winding of a synchronous machine may be on either the stator or the rotor. The difficulties of passing relatively large current at high voltages across moving contacts have made the stator wound armature the common choice for large machines. Stator-wound armature machines fall into two classes :(a) salient-pole rotor machines, and (b) non-salient-pole, or cylindrical-rotor, machines The salient-pole machine has concentrated field windings and generally is cheaper than the cylindrical-rotor machine when the speed is low, (less than 1,500 rev/min). Salient-pole alternators are generally used when the prime mover is a water turbine or a reciprocating engine. In the round or cylindrical rotor case, the held winding is placed in slots along the rotor length. The diameter is relatively small (I-1.5 m) and the machine is suitable for operation at high speeds (3000 or 3600 rpm) driven by a steam or gas turbine. Hence it is known as a turbo generator.

The frequency of the generated e.m.f, and speed are related by:
$F=np/60$
Where n is speed in rpm, and p is the number of pairs of poles.

A hydraulic turbine rotating at 50-300 rpm, depending on type. Thus needs many pole pairs to generate at normal frequencies.

Synchronous machines can be categorized into several classifications:

According to the arrangement of the field and armature windings, synchronous machines may be classified as ***rotating-armature type*** or ***rotating-field type***.

Rotating-Armature Type: The armature winding is on the rotor and the field system is on the stator.

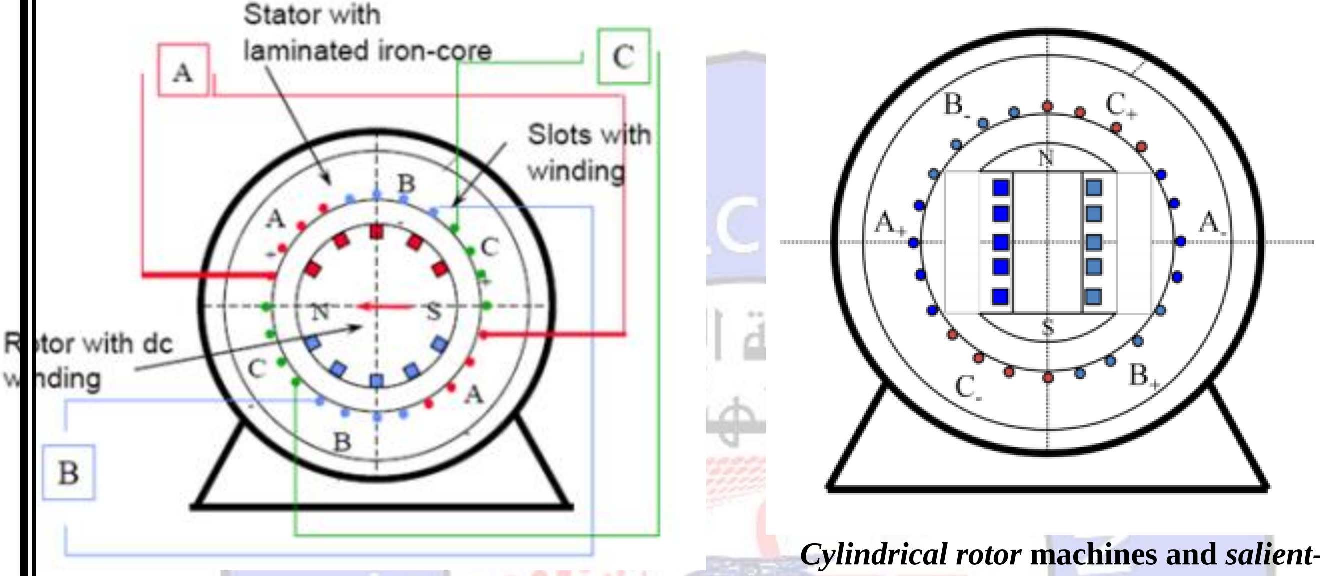
Rotating-Field Type: The armature winding is on the stator and the field system is on the rotor.

ADVANTAGES OF ROTATING FIELD OVER ROTATING ARMATURE

1. As everywhere A.C. is used, the generation level of A.C. voltage may be higher as 11 kV to 33 kV.
2. This gets induced in the armature. For stationary armature large space can be provided to accommodate large number of conductors and the insulation.
3. It is always better to protect high voltage winding from the centrifugal forces caused due to the rotation. So high voltage armature is generally kept stationary.
4. This avoids the interaction of mechanical and electrical stresses.
5. It is easier to collect larger currents at very high voltages from a stationary member than from the slip ring and brush assembly.
6. The voltage required to be supplied to the field is very low (110 V to 220 V d.c.) and hence can be easily supplied with the help of slip ring and brush assembly by keeping it rotating.
7. The problem of sparking at the slip rings can be avoided by keeping field rotating which is low voltage circuit and high voltage armature as stationary.
8. Due to low voltage level on the field side, the insulation required is less and hence field system has very low inertia.
9. It is always better to rotate low inertia system than high inertia, as efforts required to rotate low inertia system are always less.
10. Rotating field makes the overall construction very simple.
11. With simple, robust mechanical construction and low inertia of rotor, it can be driven at high speeds. So greater output can be obtained from an alternator of given size.
12. If field is rotating, to excite it by an external d.c. supply two slip rings are enough. One each for positive and negative terminals.
13. As against this, in three phase rotating armature, the minimum number of slip rings required is three and cannot be easily insulated due to high voltage levels.
14. The ventilation arrangement for high voltage side can be improved if it is kept stationary.

15. Due to all these reasons the most of the alternators in practice use rotating field type of arrangement.
16. For small voltage rating alternators rotating armature arrangement may be used.

According to the shape of the field, synchronous machines may be classified as ***cylindrical-rotor*** **(*non-salient pole*) machines** and ***salient-pole machines*.**



Cylindrical rotor **machines and** ***salient-pole machines*.**

According to the operation mod:

Synchronous Generator (Alternators)& Synchronous Motors

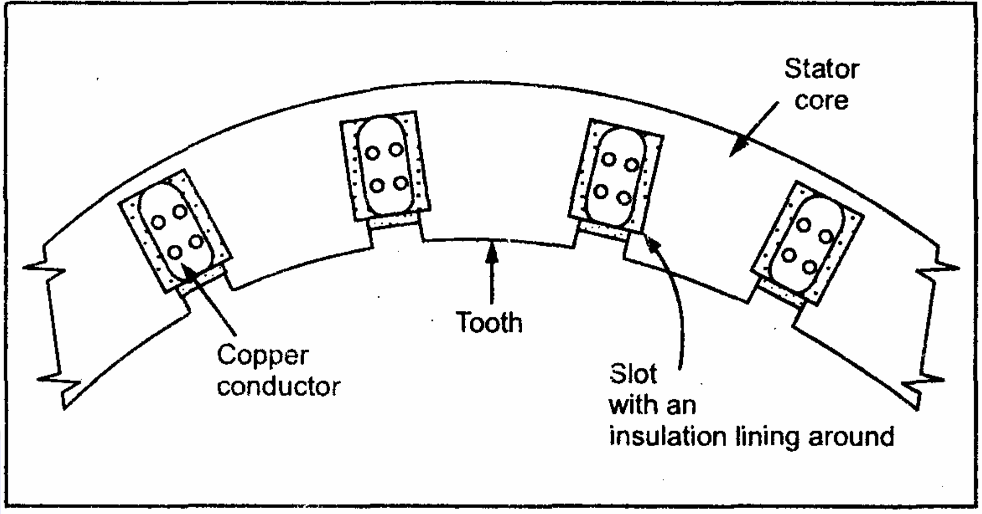
1-3 Construction of Synchronous Machines

In a synchronous generator, a DC current is applied to the rotor winding producing a rotor magnetic field. The rotor is then turned by external means producing a rotating magnetic field, which induces a 3-phase voltage within the stator winding.

STATOR

1. The stator is a stationary armature.
2. This consists of a core and the slots to hold the armature winding similar to the armature of a d.c. generator.
3. The stator core uses a laminated construction.

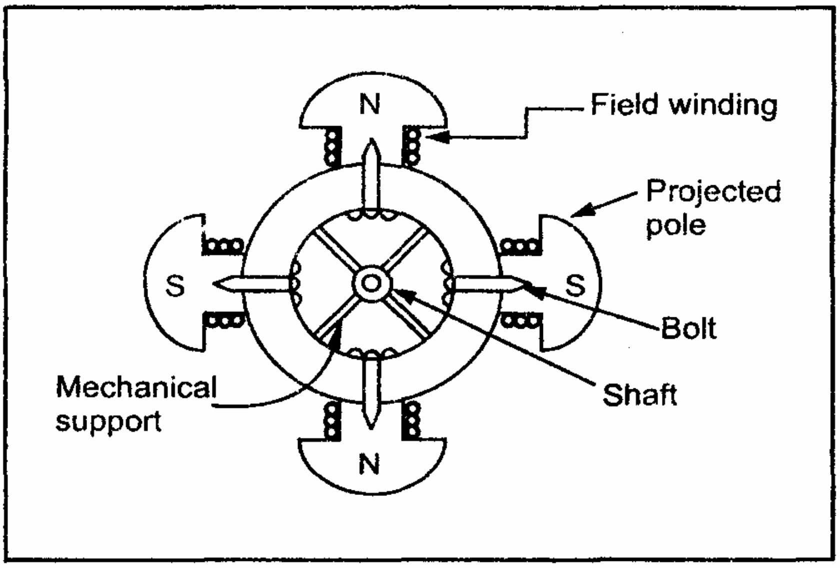
4. It is built up of special steel stampings insulated from each other with varnish or paper.
5. The laminated construction is basically to keep down eddy current losses.
6. Generally choice of material is steel to keep down hysteresis losses.
7. The entire core is fabricated in a frame steel plates.
8. The core F slots on its periphery housing the armature conductors.
9. Frame does carry any flux and serve the support to the core.
10. Ventilation is maintained with the help of holes in the frame. The section of a alternator stator is show the Figure



ROTOR

There are two types of rotors used in alternators,
i) Salient pole type ant:
ii) Smooth cylindrical type.

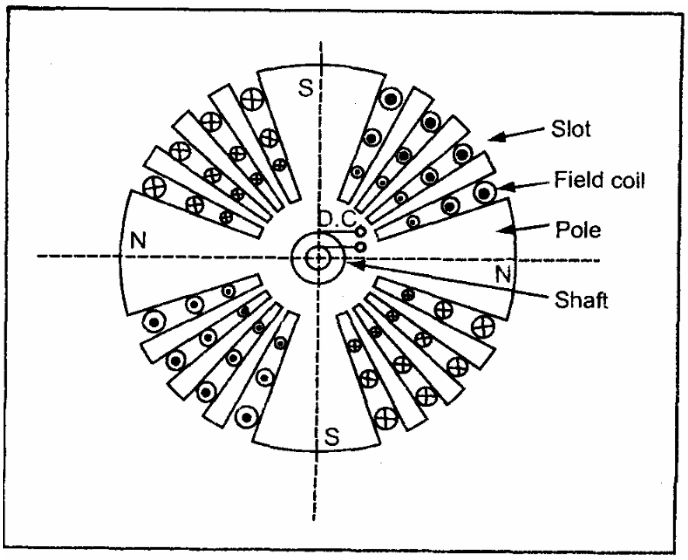
SALIENT POLE TYPE ROTOR



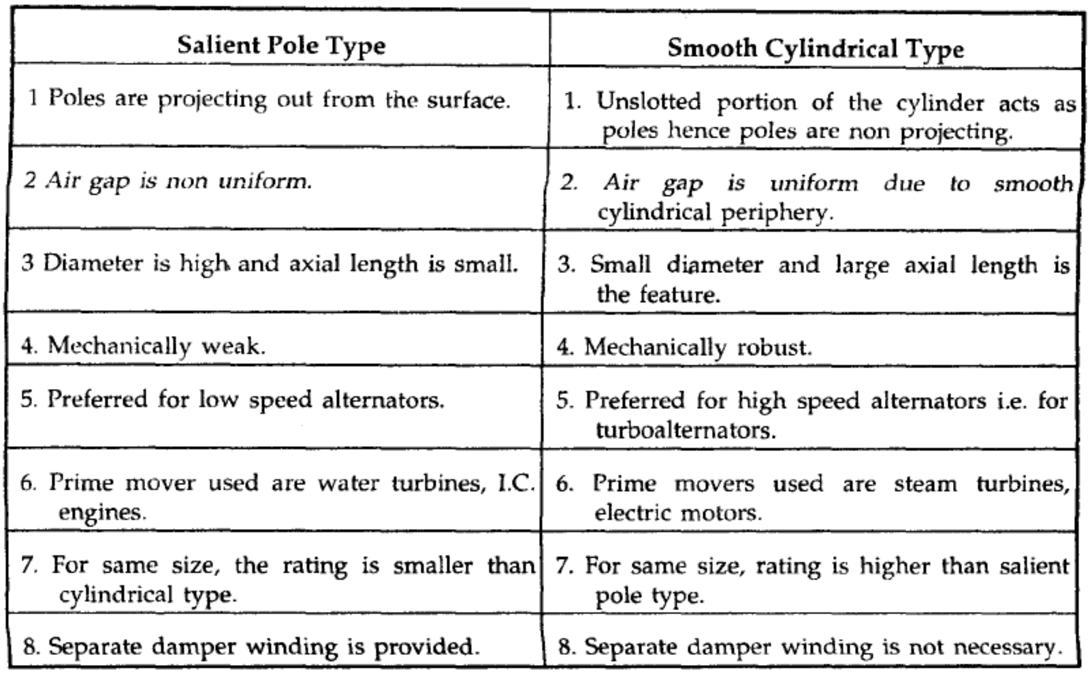
1. This is also called projected pole type as all the poles are projected out from the surface of the rotor.
2. The poles are built up of thick steel laminations.
3. The poles are bolted to t rotor as shown in the Figure.
4. The field winding is provided on the pc shoe. These rotors have large diameter and small axial lengths.
5. · The limiting factor for the size of the rotor is the centrifugal force acting on the member of the machine.
6. As mechanical strength of salient pole type is less, this is preferred for low speed alternators ranging from 125 r.p.m. to 500 r.p.m. The prime movers used to drive such rotor are generally water turbines.

SMOOTH CYLINDRICAL TYPE ROTOR

1. This is also called non salient type or non-projected pole type of rotor.
2. The rotor consists of smooth solid steel cylinder, having number of slots accommodate the field coil.
3. The slots are covered at the top with the help of steel o manganese wedge.
4. The un-slotted portions of the cylinder itself act as the poles.
5. The poles are not projecting out and the surface of the rotor is smooth which maintains uniform air gap between stator and the rotor.
6. These rotors have small diameters and large axial lengths.
7. This is to keep peripheral speed within limits.
8. The main advantage of this type is that these are mechanically very strong and thus preferred for high speed alternators ranging between 1500 to 3000 r.p.m.
9. Such high speed alternators are called ‘ turbo alternators’.
10. The prime movers used to drive such type of rotors are generally steam turbines, electric motors.



DIFFERENCE BETWEEN SALIENT AND CYLINDRICAL TYPE OF ROTOR



Salient Pole Type	Smooth Cylindrical Type
1 Poles are projecting out from the surface.	1. Unslotted portion of the cylinder acts as poles hence poles are non projecting.
2 *Air gap is non uniform.*	2. *Air gap is uniform due to smooth* cylindrical periphery.
3 Diameter is high and axial length is small.	3. Small diameter and large axial length is the feature.
4. Mechanically weak.	4. Mechanically robust.
5. Preferred for low speed alternators.	5. Preferred for high speed alternators i.e. for turboalternators.
6. Prime mover used are water turbines, I.C. engines.	6. Prime movers used are steam turbines, electric motors.
7. For same size, the rating is smaller than cylindrical type.	7. For same size, rating is higher than salient pole type.
8. Separate damper winding is provided.	8. Separate damper winding is not necessary.

- Field windings are the windings producing the main magnetic field (rotor windings)
- armature windings are the windings where the main voltage is induced (stator windings)

The rotor of a synchronous machine is a large electromagnet. The magnetic poles can be either salient (sticking out of rotor surface) or non-salient construction. Rotors are made laminated to reduce eddy current losses.

Two common approaches are used to supply a DC current to the field circuits on the rotating rotor:

1. Supply the DC power from an external DC source to the rotor by means of slip rings and brushes;

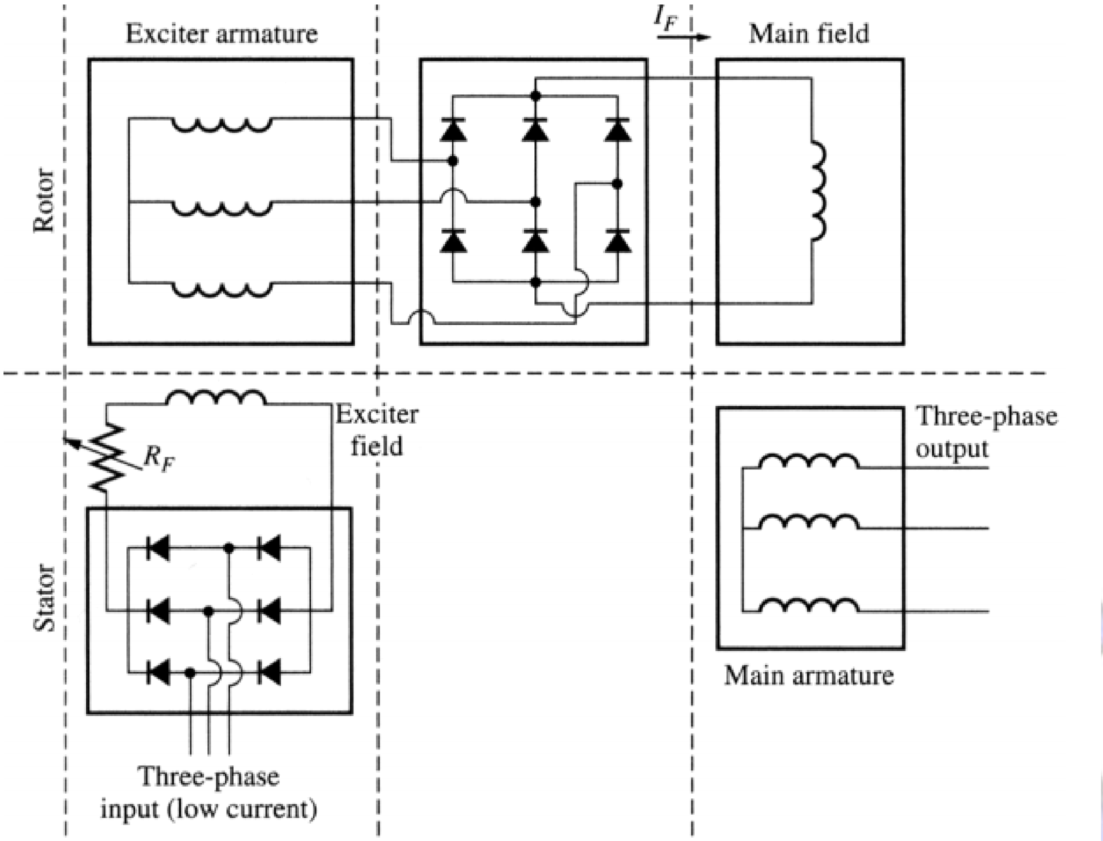
2. Supply the DC power from a special DC power source mounted directly on the shaft of the machine.

Slip rings are metal rings completely encircling the shaft of a machine but insulated from it. Graphite-like carbon brushes connected to DC terminals ride on each slip ring supplying DC voltage to field windings.

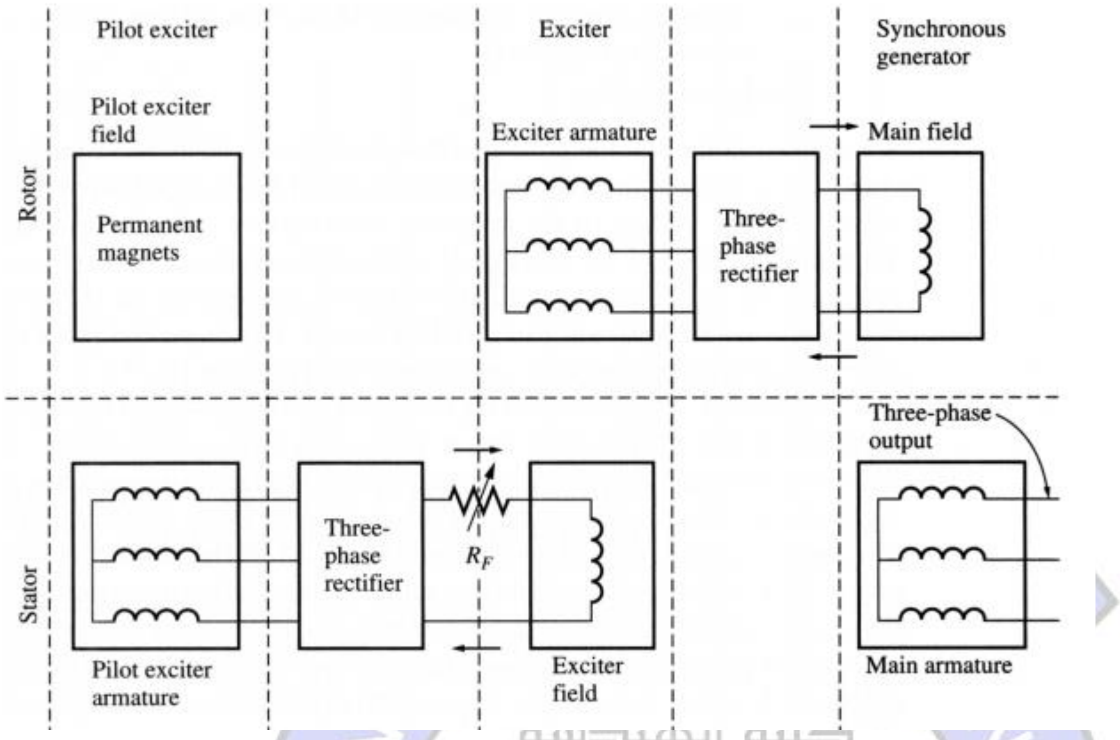
On large generators and motors, brushless exciters are used.

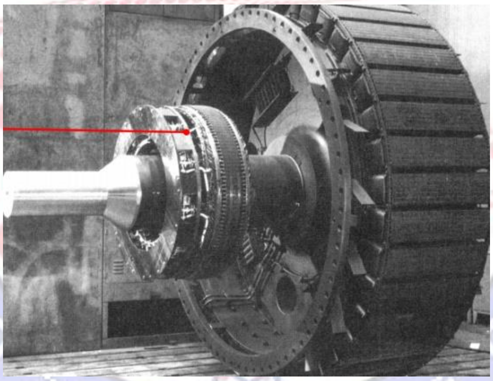
- A brushless exciter is a small AC generator whose field circuits are mounted on the stator and armature circuits are mounted on the rotor shaft.
- The exciter generator's 3-phase output is rectified to DC by a 3-phase rectifier (mounted on the shaft) and fed into the main DC field circuit.
- It is possible to adjust the field current on the main machine by controlling the small DC field current of the exciter generator (located on the stator).

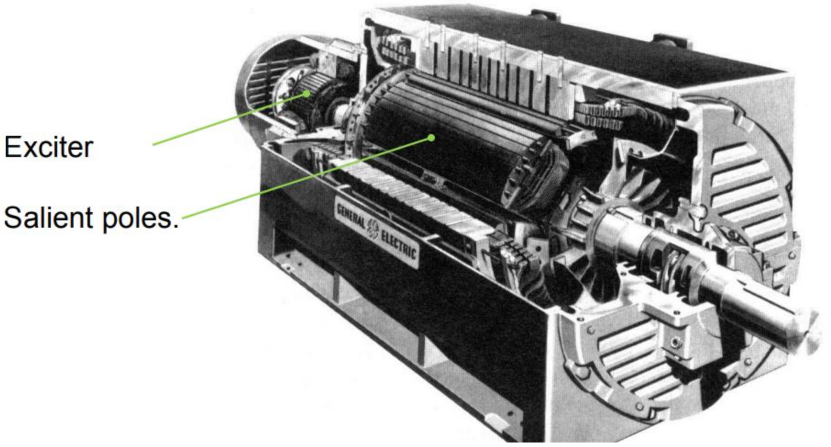
A brushless exciter: a low 3-phase current is rectified and used to supply the field circuit of the exciter (located on the stator). The output of the exciter's armature circuit (on the rotor) is rectified and used as the field current of the main machine.



To make the excitation of a generator completely independent of any external power source, a small pilot exciter is often added to the circuit. The pilot exciter is an AC generator with a permanent magnet mounted on the rotor shaft and a 3-phase winding on the stator producing the power for the field circuit of the exciter.







A rotor of large synchronous machine with a brushless exciter mounted on the same shaft

2- Synchronous Generator (Alternator):

The A.C generators or alternators are operating on the same fundamental principles of electromagnetic induction as DC generator they also consist of an armature winding and magnetic field, but there is on important difference between the two. In DC generator the armature rotates and the field system is stationary the arrangement in alternator is just the reverse of it. In their case standard construction consist of armature winding mounted on stationary element called(stator), and field winding on a rotating element called(rotor).

Then we have:

-Armature (stator), rotor (inductor)

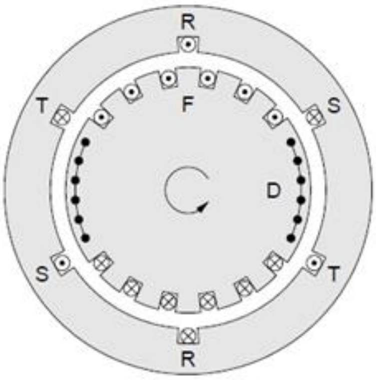
- Armature winding, excitation winding

- Damper winding.

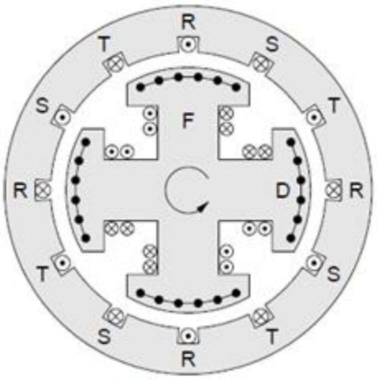
$V_o \approx 14\,KV$, $P_{ex} = 3\%P_o$, $V_{ex} = 100 - 250\,V$

-Stationary Armature and rotating inductor

Rotating Armature and stationary inductor



Round rotor



Salient pole rotor

2-1 Ventilation or Cooling of an Alternator

- The slow speed salient pole alternators are ventilated by the fan action of the salient poles which provide circulating air.

- Cylindrical rotor alternators are usually long, and the problem of air flow requires very special attention.

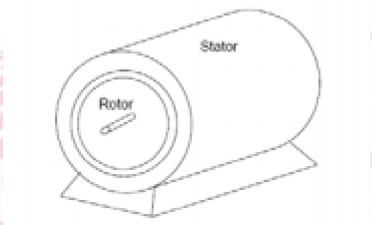
- The cooling medium, air or hydrogen is cooled by passing over pipes through which cooling water is circulated and ventilation of the alternator.

- Hydrogen is normally used as cooling medium in all the turbine-driven alternators because hydrogen provides better cooling than air and increases the efficiency and decreases the windage losses.

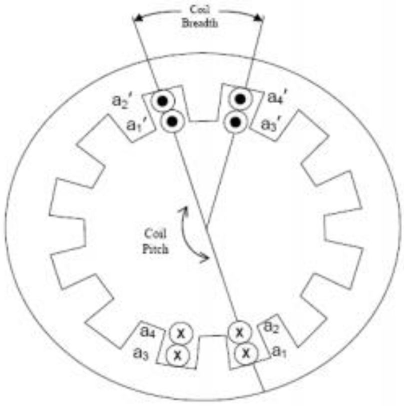
- Liquid cooling is used for the stators of cylindrical rotor generators.

2-2 Operating Principles:

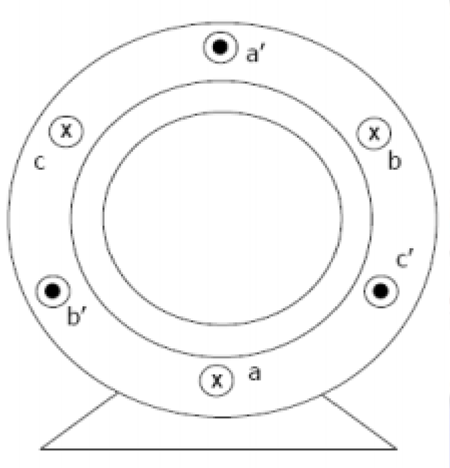
Principle of Operation A three-phase synchronous machine consists of an inner rotating cylinder called the rotor and an outer stationary housing called the stator as shown in Figure. A shaft runs through the rotor and it is balanced on bearings.



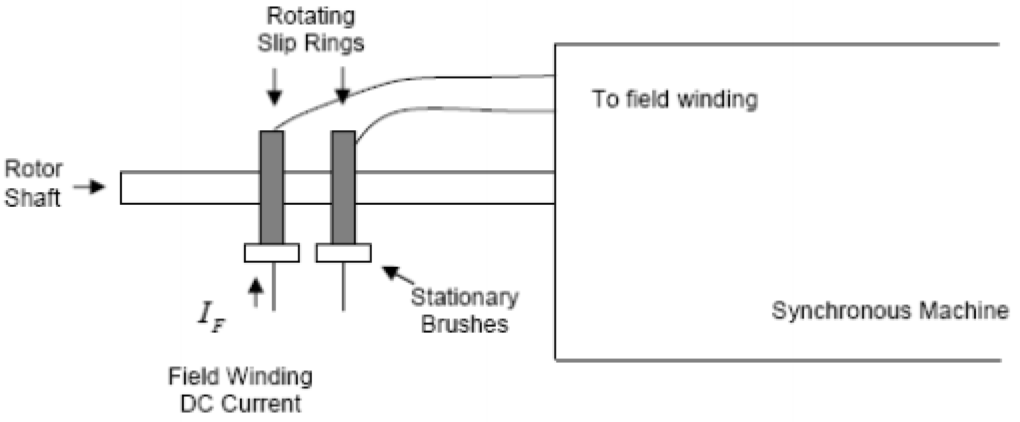
The internal periphery of a three-phase stator normally has a number of slots, the number typically being an integer multiple of six. A three-phase machine will require three identical coils of wire, each with many turns, and each coil is distributed in multiple stator slots. An example of one phase winding is shown in Figure. These windings are normally called the armature. The angular distribution of the turns is called the coil breadth. The angular distance between the sides of a given turn is termed the coil pitch. The other two-phase coils are positioned similarly about the stator periphery, with the centers of those coils spatially displaced by 120°.



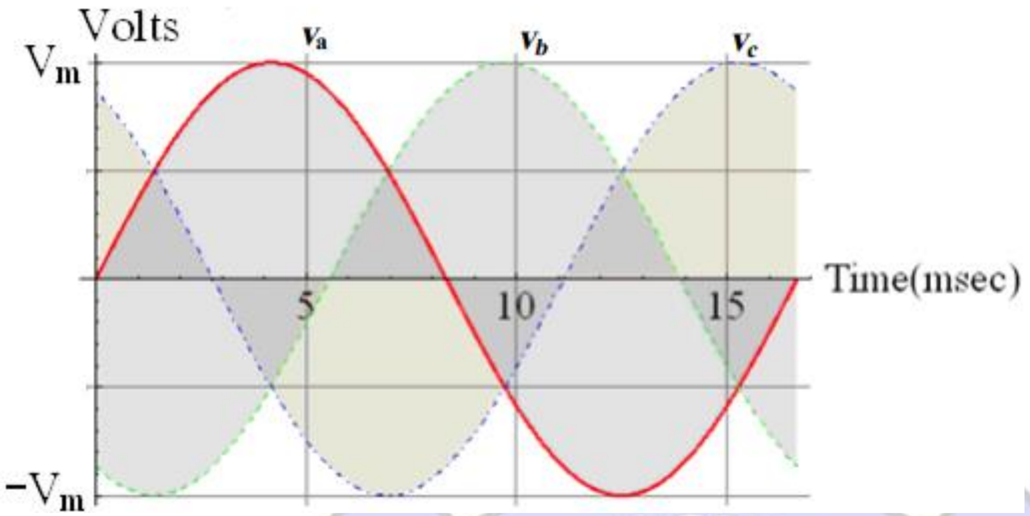
Instead of having to draw all of the slots and windings each time, we represent each distributed coil by a concentrated coil located in the center of the distribution. This is shown in Figure 5.4. The circle with a dot denotes that current is referenced out of the page while a circle with a cross indicates that current is referenced into the page. We use a, b, and c to reference the three stator phases represented in Figure.



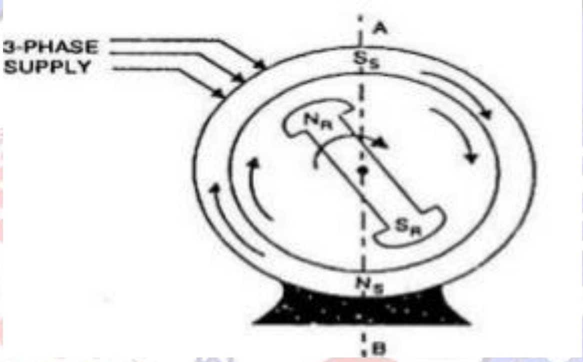
The rotor of a synchronous machine contains a winding called the field winding, which generates the magnetic field needed to generate a voltage (remember Faraday's Law). For simplicity, we will consider the case of a round rotor with a uniform air gap about the circumference of the rotor. The rotor may be slotted with the turns of the field winding distributed in those slots. The field winding will be supplied with a DC current. You say, "Wait a second, the field winding is on the rotor, and the rotor is spinning. How can we supply DC current to something that is moving?" The simplest solution to this dilemma is to use slip rings and brushes as illustrated in Figure. Note that the end connections of the field winding are tied to two copper rings mounted on the rotor shaft. Stationary carbon brushes are then made to ride upon the rings. A stationary DC voltage source is then applied to the brushes allowing DC current to flow through the field winding. Since the brushes are not commutating (i.e., reversing the current) coils as in a DC machine, the wear and maintenance requirements are not as intensive.



The DC current flowing in the field winding will set up a magnetic field on the rotor (think North and South poles). The prime mover (mechanical engine) will then spin the rotor at what we will soon refer to as synchronous speed. The magnetic field sweeping past the stationary stator coils will induce voltages. This phenomenon is described by Faraday’s law, and was present as the back EMF in the DC motors you studied previously. Since the phase coils are spatially displaced, the induced voltages will be time displaced and will constitute a balanced set (i.e., same frequency, equal amplitude, and 120° displaced in phase). The voltage produced by each phase coil is shown in Figure. If we imagine that the rotor magnetic field moves past the “a” stator phase first, we would expect a strong induced voltage for the a-phase. As the rotor turns and moves its magnetic field past the b and c coils, those coils would also show a surge in voltage respectively. The sequence of voltages shown in the figure is termed the abc-phase sequence since the a-phase takes its peak first, then the b-phase and finally the c-phase. Note that the voltages all have the same frequency and equal amplitude but are displaced from each other by 120°. (As the rotor turns and moves past the a’, b’ and c’, the negative voltage peaks occur.)



$V_e \rightarrow I_e \rightarrow F_e \rightarrow \emptyset_e$



At no load $V_1 = E_e = E_o$

n_r-prime mover speed

$F_i = \frac{P_{nr}}{60}$ emf frequency

At load $I_1 = I_a \rightarrow F_a \rightarrow \emptyset_a$ rotating with (n_s)

$$n_s = \frac{60\ F_1}{P} = \frac{60}{P}\left(\frac{P_{nr}}{60}\right) = n_r$$

$$n_s = n_r$$

2-3 Constructional Element

1-Armature: hollow cylinder assembled from silicon lamination, with slots to contain the windings open slots with parallel sides with double layer lap winding for voltage up to 11 KV

Semi open slots with single layer lap winding for voltage up to 33 KV

2-Salient poles or non-salient (cylindrical) rotors.

3-Damper winding, separate or complete.

It is starting winding in the motor and in the alternators:

a- Weakening the negative effect of unbalance loads.

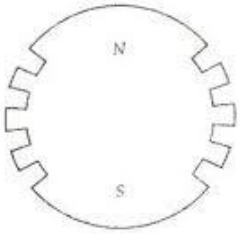
b-damps the rotor swinging due to sudden load change.

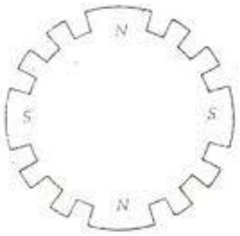
4- Excitation – from external source (250v) through the slip rings or brushes (diodes). Using (AVR) usually the winding from copper stripes

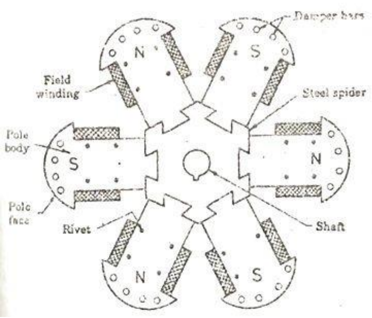
5-Cooling-close system used for cylindrical machines up to 25 μw. The advantage is:

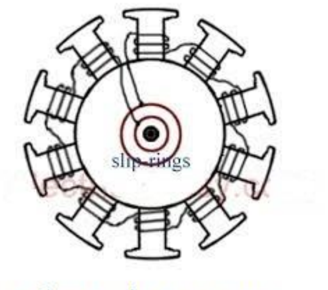
a-It is density is 10% of the air

b-high thermal conductivity, which loads to 30% increase in loading.









Salient pole type rotor

2-4 Armature Reaction:

Armature reaction is the effect of armature flux on the mean field flux. In the case of alternator, the power factor of the load has a considerable effect on the armature reaction

1-Direct Axis, Quadrature Axis

$V_e \rightarrow I_e \rightarrow F_e \rightarrow \emptyset_e$

At $n_r \rightarrow E_{oA}, E_{oB}, E_{oC}$

At load $\rightarrow I_a \rightarrow F_a \rightarrow \emptyset_a$

$F_a = 1.35\, I_a\, Kw\, T_s$

Ψ- phase shift between I_a and E_o

From $\emptyset_e$ and $\emptyset_a$ result $\emptyset_g$- air gap flux

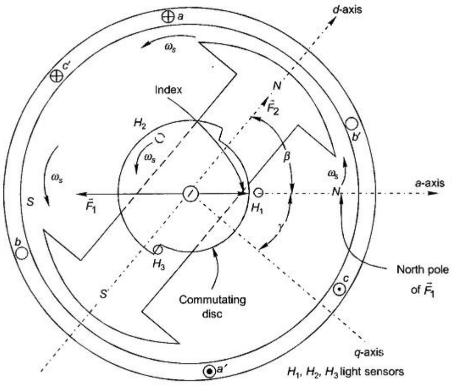
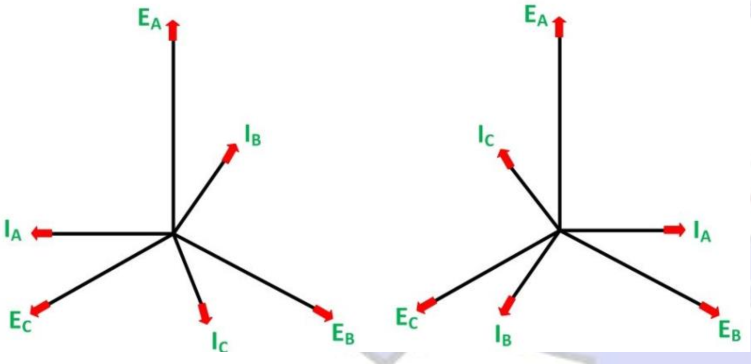


Fig. 8.84 *Brushless dc motor arrangement of sensors; 120° elect sensor code switching from 101 to 100*



2-Pure inductive load(R=0) then Ψ=90

$$I_A = 0 \quad , I_B = -\frac{\sqrt{3}}{2} \times I_m \quad , \quad I_C = \frac{\sqrt{3}}{2} \times I_m$$

Direct armature reaction and demagnetizing armature reaction.

$$\mathbf{I_a} \uparrow, \boldsymbol{\emptyset_a} \uparrow \rightarrow \boldsymbol{\emptyset_g} \downarrow, \boldsymbol{E_g} \downarrow$$

4-Pure capacitive load

Ψ=-90°

$\mathbf{I_A = 0}$, $\boldsymbol{I_B = \frac{\sqrt{3}}{2} \times I_m}$, $\boldsymbol{I_C = -\frac{\sqrt{3}}{2} \times I_m}$

Direct armature reaction magnetizing A.R

$\mathrm{I_a} \uparrow, \emptyset_a \uparrow \rightarrow \emptyset_g \uparrow \rightarrow E_g \uparrow$

4-Pure Ohmic (resistive) load.

$\Psi=0^{o}$,, $\mathrm{I_a}$ in phase with E_a

$\mathrm{I_A} = \mathrm{I_m}$, $\mathrm{I_B}$, $\mathrm{I_C} = -\frac{I_m}{2}$

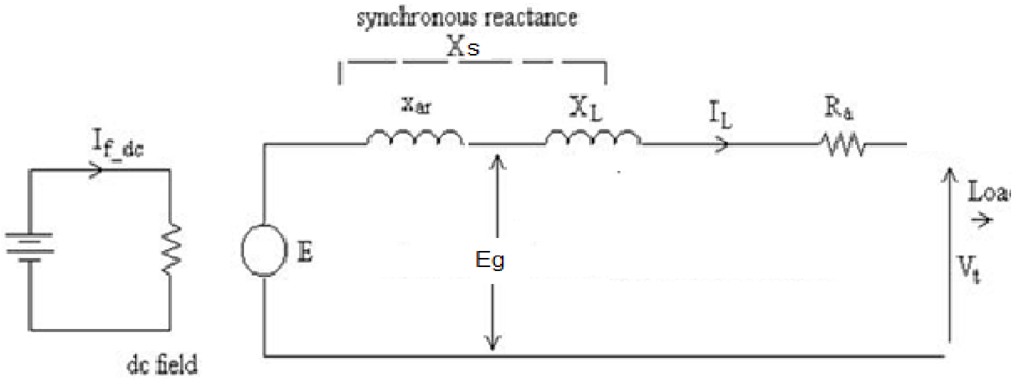
2-5 Phasor Diagram for Cylindrical Rotor Generator

1-saturated cylindrical rotor SM

$R_1, X_L, I_1, V_1, \cos\varphi$ and OCC

$F_e = \mathrm{F_g} - \mathrm{F_a}$, $F_e = I_e T_e$

$\mathrm{F_a} = 1.35\, I_1\, K w T_s$



Per Phase Equivalent Circuit

-voltage regulation $(VR\%)$

$$VR\% = \frac{E_o - V_r}{V_r} \times 100\%$$

$$E_o = \mathrm{E_g} + (-E_a)$$

Advantages:

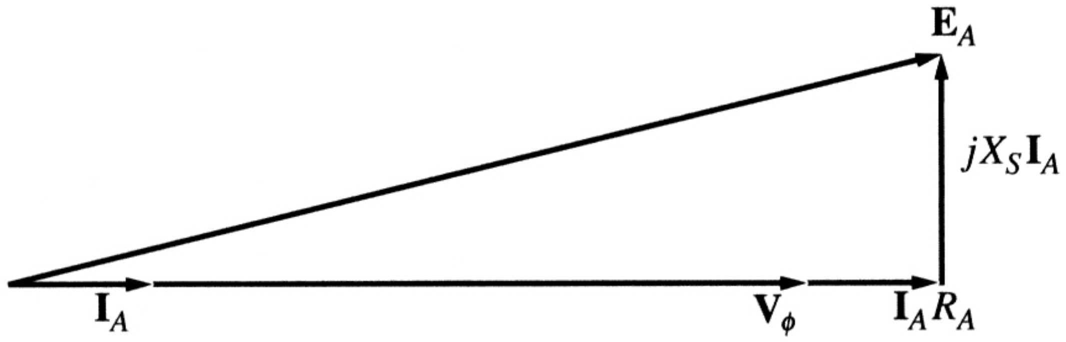
- Simple no load tests (for obtaining OCC and SCC) are to be conducted
- Calculation procedure is much simpler

Disadvantages:

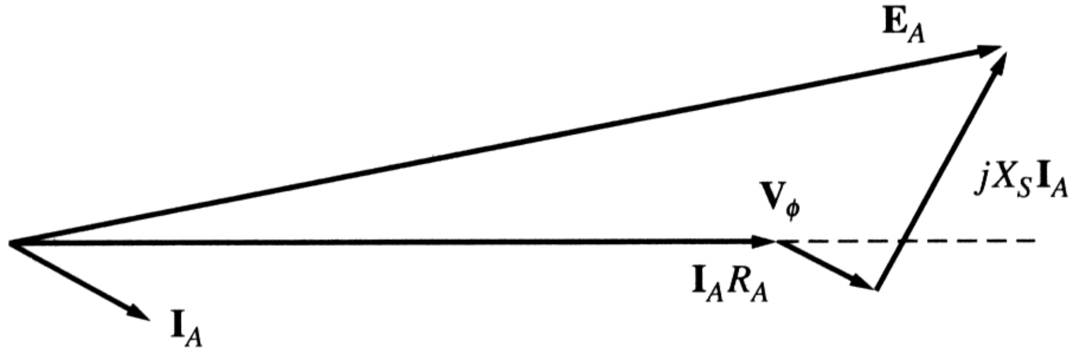
- The value of voltage regulation obtained by this method is always higher than the actual value.

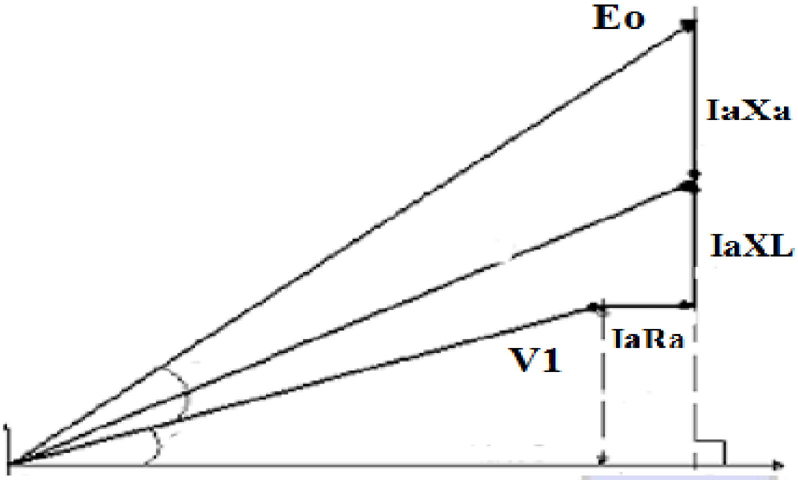
$$E_a = -^{j} I_a X_a \quad , \quad E_g = V_1 + I_1(R_a + {}^{j} X_L$$
$$\mathrm{E_o} = \mathrm{V_1} + \mathrm{I_a R_a} + jI_a(X_L + X_a) = V_1 + I_a(R_a + jX_s$$

For unity power factor: pure resistive load



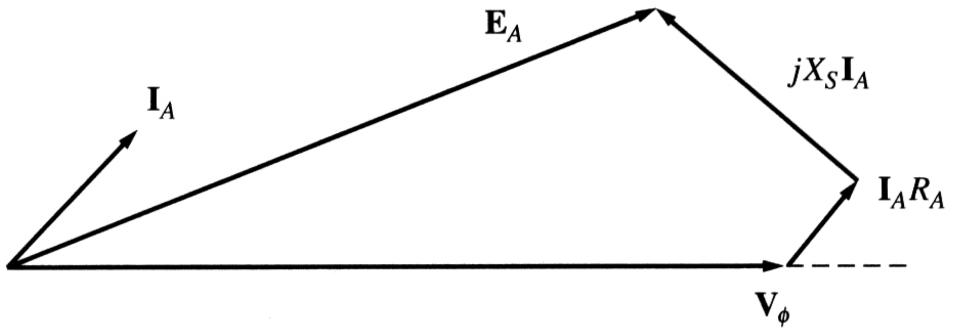
For lagging power factor: inductive load.





$$E_o = \sqrt{v_1\ \cos\theta_1 + I_aR_a)^2 + (V_1 \sin\theta_1 + I_aX_s)^2}$$

For leading: capacitive load



$$E_o = \sqrt{v_1 \ \cos\theta_1 + I_a R_a)^2 + (V_1 \sin\theta_1 - I_a X_s)^2}$$

2-6 Energy Diagram:

$$\sum P_{losses} = P_1 - P_{2-}$$

$$= P_{fe} + P_{cua} + P_{cue} + P_{fw} + P_{add}$$

P_{fe}**: Iron Losses**

P_{cua}**: Cupper Losses in Armature**

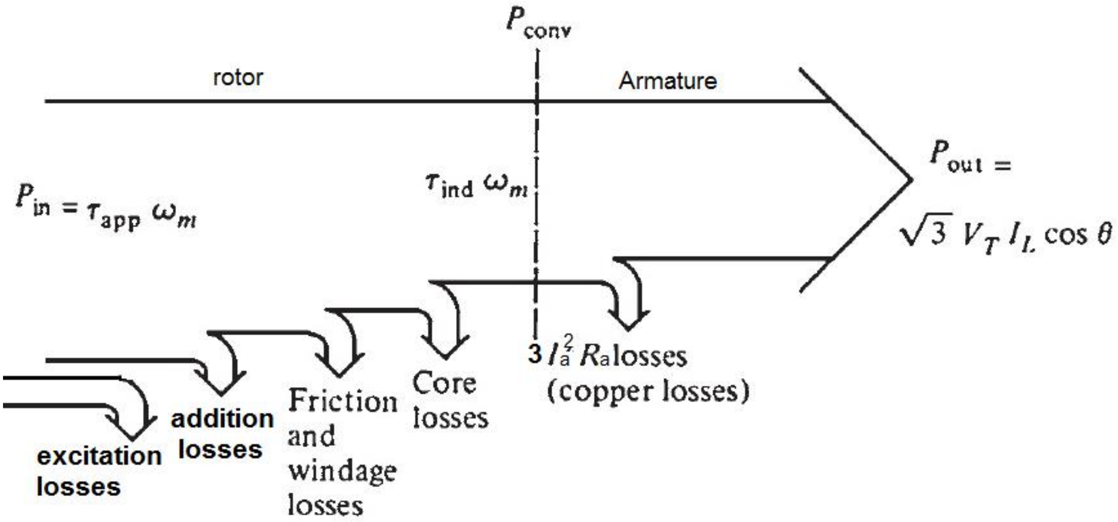
P_{cue}**=Cupper Losses in Excitation Winding**

P_{fw}**: Friction and Winding Losses**

P_{add}**: Addition Losses**

$$P_{cua} = 3I_a^2 \ R_a$$

$$P_{cue} = I_e^2 \ R_e$$



The power flow diagram of a synchronous generator.

$P_{add} = 0.5\%\, P_1$, $P_{fw} = 50\%\, P_{losses}$

$P_{em} = P_2 + P_{cua}$ or $P_2 = P_{em} - P_{cua}$

:For separate excitation then

$P_i = P_1 + P_{ex}$

For self-excitation, then:

$P_o = P_2 - P_{ex}$

ŋ%=$\frac{P_o}{P_i} \times 100\% = \frac{P_o}{P_o+P_{fw}+P_{fe}+P_{cua}+P_{ad}} \times 100\%$

2-7 Measuring parameters of synchronous generator model

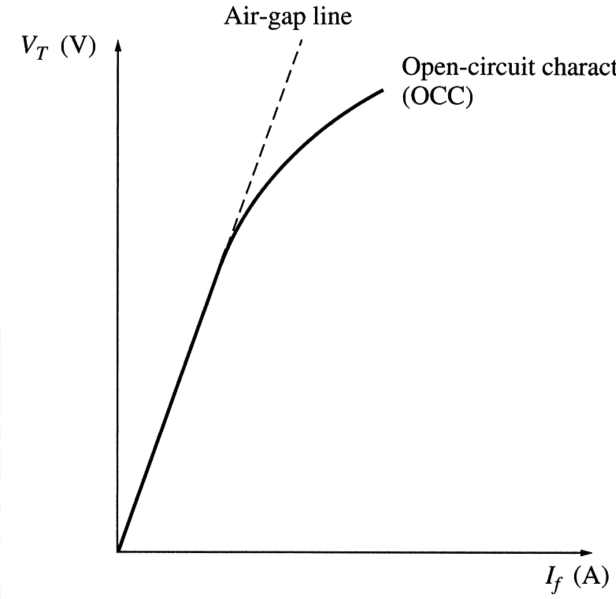
The three quantities must be determined in order to describe the generator model:
1.The relationship between field current and flux (and therefore between the field current *IF* and the internal generated voltage *EA*);
2.The synchronous reactance;
3.The armature resistance.

Open circuit Test:

The generator is rotated at the rated speed,
•all the terminals are disconnected from loads,

•the field current is set to zero first.

•Next, the field current is increased in steps and the phase voltage (whish is equal to the internal generated voltage *EA* since the armature current is zero) is measured.
Since the unsaturated core of the machine has a reluctance thousands times lower than the reluctance of the air-gap, the resulting flux increases linearly first. When the saturation is reached, the core reluctance greatly increases causing the flux to increase much slower with the increase of the mmf.



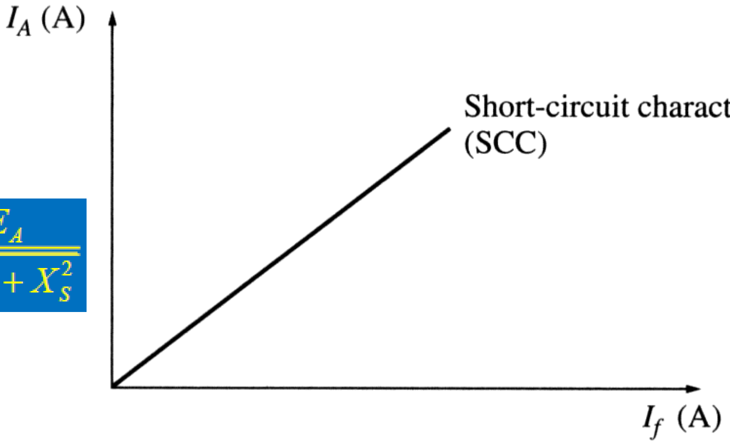
Short Circuit Test

In here,

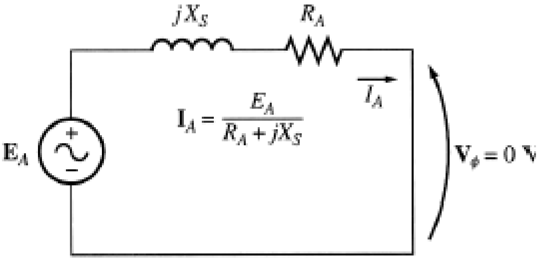
- The generator is rotated at the rated speed, with the field current is set to zero first, and all the terminals are short-circuited through ammeters.
- Next, the field current is increased in steps and the armature current *IA* is measured as the field current is increased. The plot of armature current (or line current) vs. the field current is the short-circuit characteristic (SCC) of the generator.

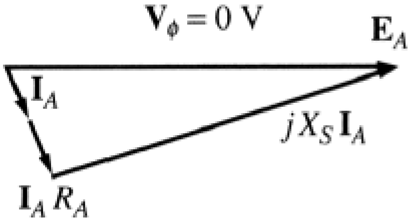
The SCC is a straight line since, for the short-circuited terminals, the magnitude of the armature current is

$$I_A = \frac{E_A}{\sqrt{R_A^2 + X_S^2}}$$



The equivalent generator’s circuit during SC





An approximate method to determine the synchronous reactance *XS* at a given field current:

1. Get the internal generated voltage *EA* from the OCC at that field current.
2. Get the short-circuit current *IA,SC* at that field current from the SCC.
3. Find *XS* from

$$X_S \approx \frac{E_A}{I_{A,SC}}$$

Since the internal machine impedance is

$$Z_S = \sqrt{R_A^2 + X_S^2} = \frac{E_A}{I_{A,SC}} \approx X_S \qquad \{\text{since } X_S \approx R_A\}$$

2-8 Parallel operation of alternator:

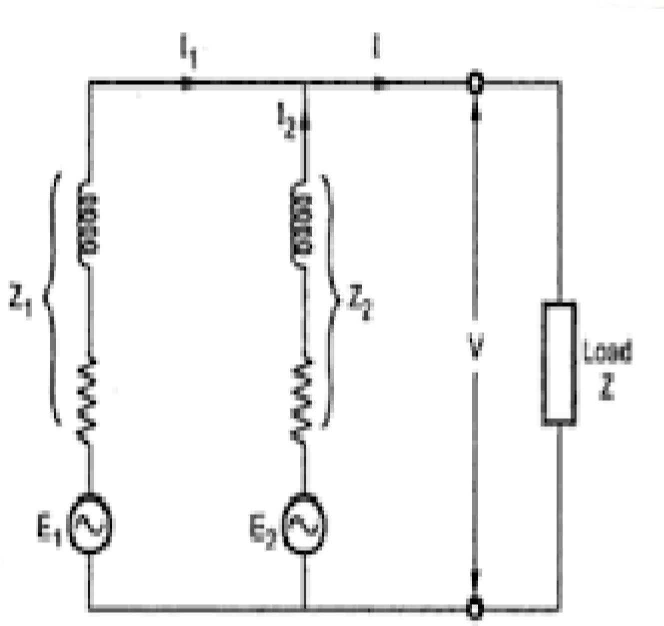
The operation of connecting an alternator in parallel with another alternator or with common bus bars is known as a synchronizing. Generally, alternators are used in power system where they are in parallel with many other alternators. It means that the alternator is connected to alive system of constant voltage and constant frequency.

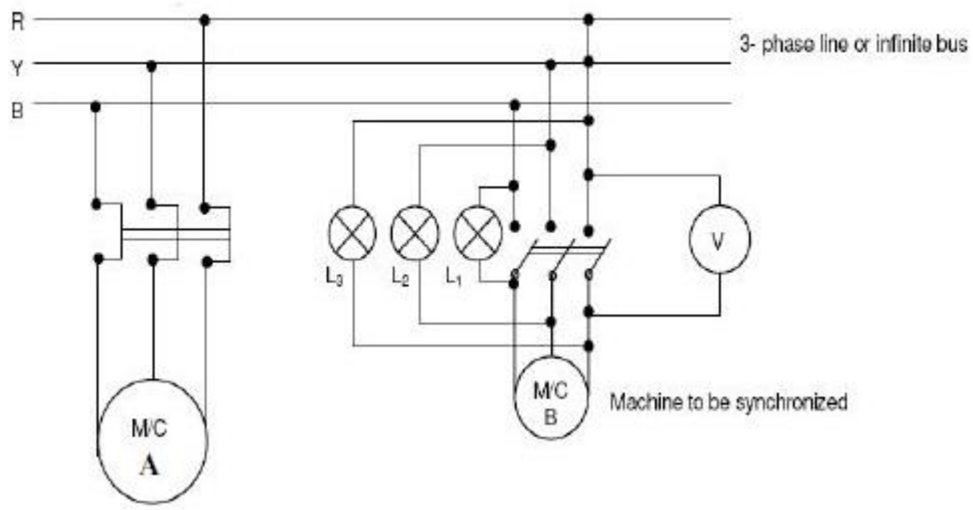
There are three conditions must be satisfied when connect the alternators in parallel:

1-The terminal voltage of incoming alternator must be the same as bus- bar voltage

2-the speed f the incoming machine must be such that its frequency ($f = \frac{P_o}{60}$) equals bus-bar frequency.

3-the phase of the alternator voltage must be identical with the phase of the bus-bar voltage.





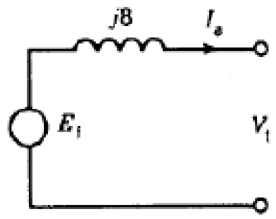
EXAMPLE

A 3ϕ, 5 kVA, 208 V, four-pole, 60 Hz, star-connected synchronous machine has negligible stator winding resistance and a synchronous reactance of 8 ohms per phase at rated terminal voltage.

Determine the excitation voltage and the power angle when the machine is delivering rated kVA at 0.8 PF lagging. Draw the phasor diagram for this condition.

Solution

The per-phase equivalent circuit for the synchronous generator



$$V_t = \frac{208}{\sqrt{3}} = 120 \text{ V/phase}$$

Stator current at rated kVA:

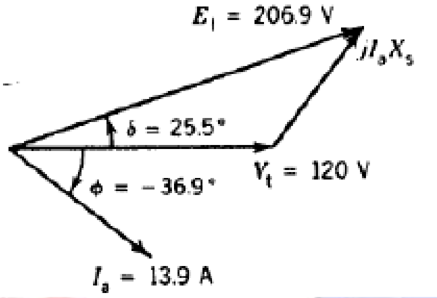
$$I_a = \frac{5000}{\sqrt{3} \times 208} = 13.9 \text{ A}$$

$\phi = -36.9^\circ$ for lagging pf of 0.8

$$\begin{aligned} E_f &= V_t\angle 0^\circ + I_a jX_s \\ &= 120\angle 0^\circ + 13.9\angle -36.9^\circ \cdot 8\angle 90^\circ \\ &= 206.9\angle 25.5^\circ \end{aligned}$$

Excitation voltage $E_f = 206.9$ V/phase

Power angle $\delta = +25.5^\circ$



H.W

Q1) A four pole, three-phase synchronous generator is rated 250 MVA, its terminal voltage is 24 kV, the synchronous reactance is: 125%.

- Calculate the synchronous reactance in ohm.
- Calculate the rated current and the line to ground terminal voltage.
- Draw the equivalent circuit.
- Calculate the induced voltage, E_f, at rated load and pf = 0.8 lag.

(Ans: $X_{syn}=2.88\Omega$, $I_g=6.01\angle -36.87^{o}$KA, $E_{gn}=27.93\angle 29.74$KV)

Q2) A3-phase, star connected alternator is rated at 1600Kva, 3500V. the armature effective resistance and synchronous reactance are 1.5Ω and 30Ω respectively per phase. Calculate the percentage regulation for a load of 1280Kw at power factor of a) 0.8 lagging b) 0.8 leading c) unity.

(Ans. 18.6, -11.99, 3.227)

Q3) A 3-phase, 120kV, 1.5 MVA, alternator, its star connected armature winding has 1Ω effective resistance and 10Ω synchronous reactance per phase find VR% at full load at: a) unity power factor, b) 0.8 lagging, c) 0.8 leading. (Ans. 1.56%, 7.36%, -5%)

Q4) A 3-phase, 0.8MVA, 3.3Kv, 50Kz, SG, its armature winding star connected and its iron and mechanical losses are 20kW, having 0.5Ω/phase armature resistance. When a 150V is applied to the excitation winding Ie=100A at cosφ=1 and Ie=120A at cosφ=0.8lagging. find the efficiency at rated load and unity power factor and 0.8 lagging. (Ans. 9205%, 90.46%)

Q5) A 3-phase, delta connected, 15MVA, 10kV alternator has an armature resistance of 0.4Ω/phase and syncronouse reactance of 1.2Ω/phase. Find the full load voltage regulation at 0.8 power factor leading and lagging. (Ans. -1.82%, 5.26%)

3- Synchronous Motors

3-1 Introduction:

A synchronous motor is electrically identical with an alternator or A.C generator. in fact, a given synchronous machine may be used, at least theoretically, as an alternator, when driven mechanically or as a motor then driven electrically just as in the case of D.C machines. Must synchronous motor be rated between 150 Kw to 15 µw and run at speeds ranging from 150 to 1800 r.p.m.

Some characteristic features of a syn. Motor

Are worth noting:

1-It runs either at syn. Speed or not at all, While running it maintains constant speed.

The only way to change speed is to vary the supply frequency, $N_s = 60\frac{f}{P}$

2-It is not inherently self-starting. It has to be run up to sync. Speed or near it, by some means, before it can be synchronized to the supply

3-Mainly salient-pole motors, for low and medium rang speed.

- Single- phase and 3-phase motor ʎ or △

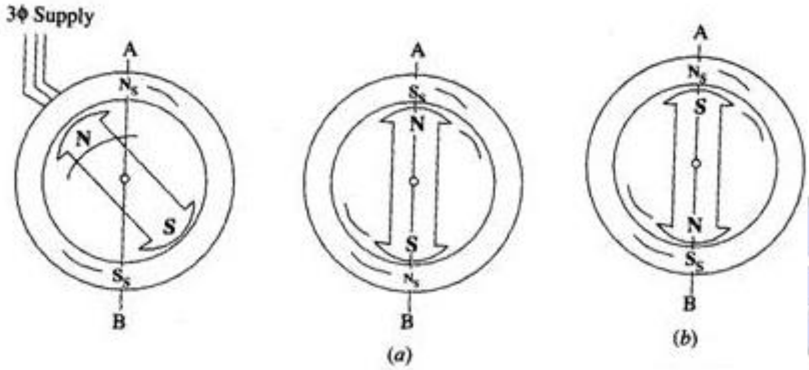
Conn. Arm. Winding

4-Damper or starting winding. Air gap length is lower SG

$\frac{I_s}{I_r} = 4 - 5$, $\frac{T_s}{T_r} = 2 - 3$ lagging reaction P.F

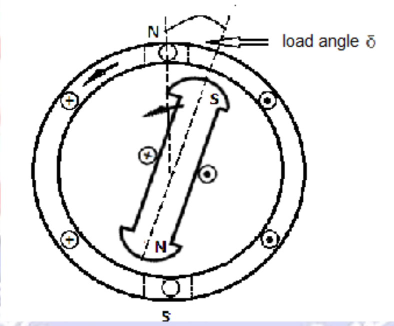
3-2 Motor Operation:

When a 3-∅ winding is fed by a 3-∅ supply, then magnetic flux of constant magnitude but rating at syn. speed, is produced. Consider a two-pole stator of fig. below. In which are shown two poles at stator. Marked N_s and S_s rotating at syn. speed, say in clockwise direction. With the rotor position as shown, suppose. The stator poles are at that instant situated at points A and B. The two similar poles N (of rotor) and N_s of stator as well as S and S_s will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction, but half a period later, stator poles, having rotated around. Interchange their position, it will be rotate with clockwise.



Load angle (pole axis E_o logging relative to the flux axis v)

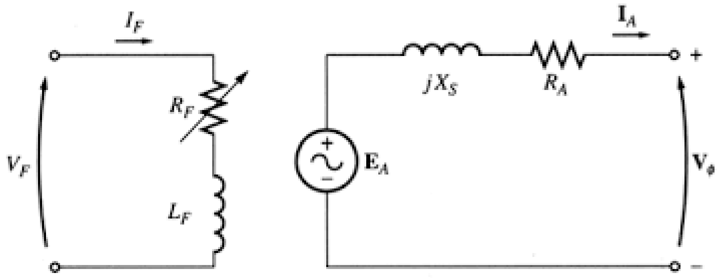
load↑ → δ ↑ , In generator $+\delta$, in motor $\delta -$



3-3 Equivalent circuit of syn. motor

The equivalent circuit model for one armature phase of a cylindrical rotor syn. motor is shown in fig.1, it is seen that the phase applied voltage(v) is the vector sum of reversed back emf E_o and the impedance drop I_1 , Z_s . In other words,

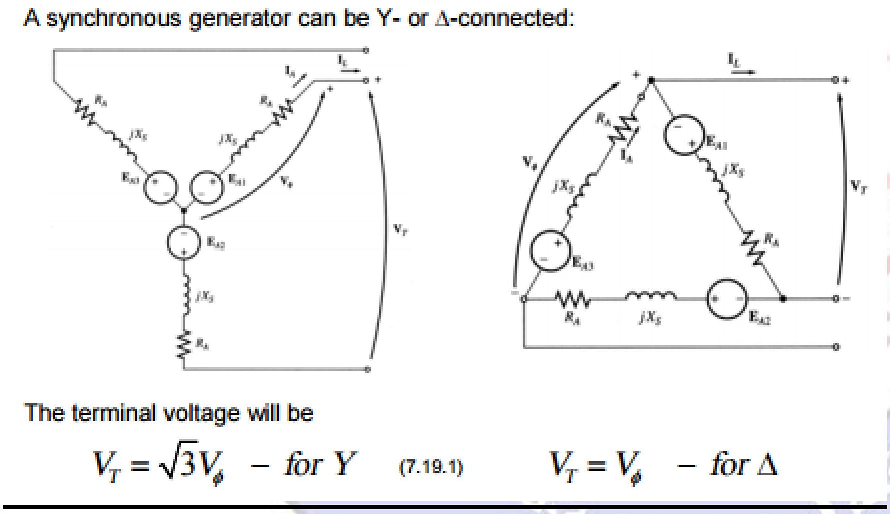
$V = (-E_o + I_1 Z_s)$. The angle δ between the phase for (V) and (E_o) is called the load angle of syn. motor.



When $R_a = 0$

$$E_o = V - j\, I_a\, X_s$$

$$\delta = \sin^{-1}\left(\frac{X_s\, P}{3\, V E_o}\right)$$



A synchronous generator can be Y- or Δ-connected:

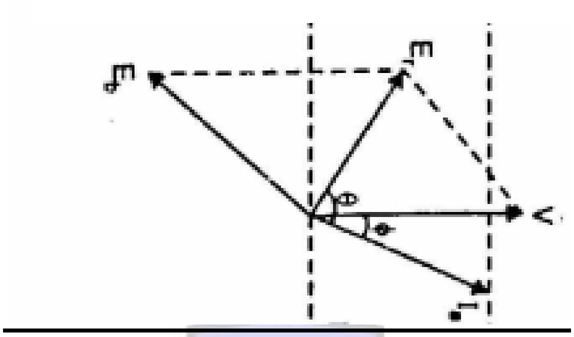
The terminal voltage will be

$$V_T = \sqrt{3} V_\phi \quad - \text{ for } Y \qquad (7.19.1) \qquad V_T = V_\phi \quad - \text{ for } \Delta$$

3-4 Phasor Diagrams:

In the syn. motor, a back e.m.f is set up the armature (stator) by the rotor flux which opposes the applied voltage(v).This back e.m.f depends on rotor excitation only .The net voltage in armature (stator) is the vector difference of V and E_o. Armature current is obtained by dividing the vector difference of voltage by armature impedance. The simplified phaser diagram is represented as follow: a-for lagging power factor b-for leading power factor

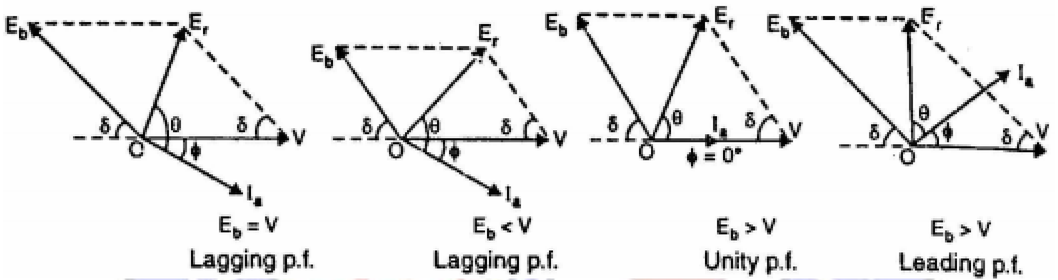
a-For lagging power factor:



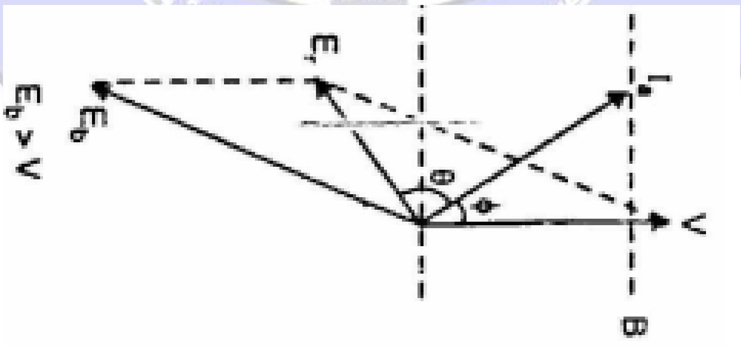
$$E_o^2 = (V + E_r \sin\varphi)^2 + (E_r \cos\varphi)^2$$

$$E_o^2 = V^2 + E_r^2 - 2VE_r \cos\gamma$$

$$\gamma = \theta - \varphi$$



b-for leading power factor:



$$E_o^2 = (V + E_r \sin\varphi)^2 + (E_r \cos\varphi)^2$$

$$E_o^2 = V^2 + E_r^2 - 2VE_r \cos\gamma \qquad ,\gamma = \theta + \varphi$$

Note:: If $R_a = 0$ $, Z_s = X_s$ $, E_r = I_1 X_s$

The power supplied to the motor is:

$$P_1 = \sqrt{3}\, V_s\, I_1 \ \cos\varphi_1$$

The electromagnetic power supplied to the rotor is:

$$P_{em} = P_1 - P_{cu1}$$

$$= \sqrt{3}\, V_s\, I_1 \ \cos\varphi_1 - 3\, I_1^2\, R_1$$

$$E_r = \sqrt{(V - E_o \cos\delta)^2 + (E_o \sin\delta)^2}$$

$$\gamma = \tan^{-1}\frac{E_o \ \sin\delta}{V - E_o \ \cos\delta} \qquad , \quad \delta = \sin^{-1}\left(\frac{X_s \ P}{3VE_o}\right)$$

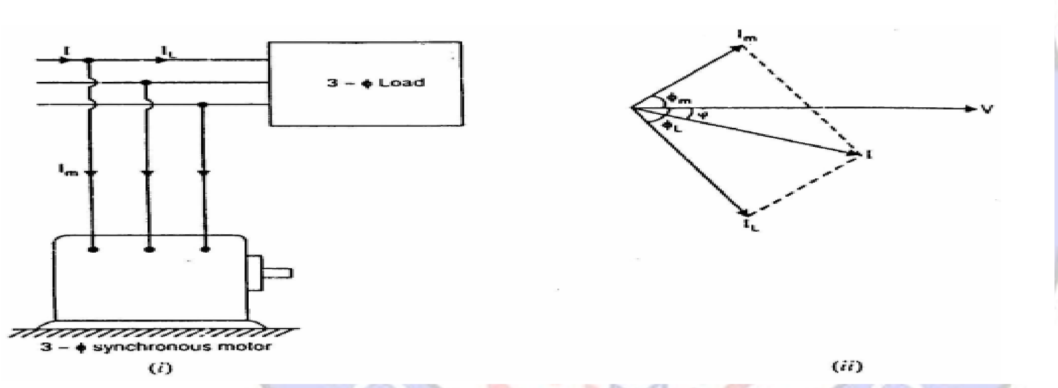
$$P = \frac{3VE_o}{X_s} \times \sin\delta$$

3-5 Synchronous Condenser:

A synchronous motor takes a leading current when over-excited and, therefore, behaves as a capacitor. An over-excited synchronous motor running on no-load in known as synchronous condenser. When such a machine is connected in parallel with induction motors or other devices that operate at low lagging power factor, the leading kVAR supplied by the synchronous condenser partly neutralizes the lagging reactive kVAR of the loads. Consequently, the power factor of the system is improved. Fig. (11.14) shows the power factor improvement by synchronous condenser method. The 3 - Øload takes current I_L at low lagging power factor cos$\emptyset_L$. The synchronous condenser takes a current I_m which leads the voltage by an angle $\emptyset_m$. The resultant current I is the vector sum of I_m and I_L and lags behind the voltage by an angle Ø. It is clear that Ø is less than $\emptyset_L$ so that cos Ø is greater than cos $\emptyset_L$. Thus the power factor is increased from cos $\emptyset_L$ to cos Ø. Synchronous condensers are generally used at major bulk supply substations for power factor improvement.

Advantages

(i) By varying the field excitation, the magnitude of current drawn by the motor can be changed by any amount. This helps in achieving stepless control of power factor.
(ii) The motor windings have high thermal stability to short circuit currents.
(iii) The faults can be removed easily



Disadvantages

(i) There are considerable losses in the motor.
(ii) The maintenance cost is high.
(iii) It produces noise.

(iv) Except in sizes above 500 KVA, the cost is greater than that of static capacitors of the same rating.

(v) As a synchronous motor has no self-starting torque, then-fore, an auxiliary equipment has to be provided for this purpose.

Applications of Synchronous Motors

(i) Synchronous motors are particularly attractive for low speeds (< 300 r.p.m.) because the power factor can always be adjusted to unity and efficiency is high.
(ii) Overexcited synchronous motors can be used to improve the power factor of a plant while carrying their rated loads.
(iii) They are used to improve the voltage regulation of transmission lines.

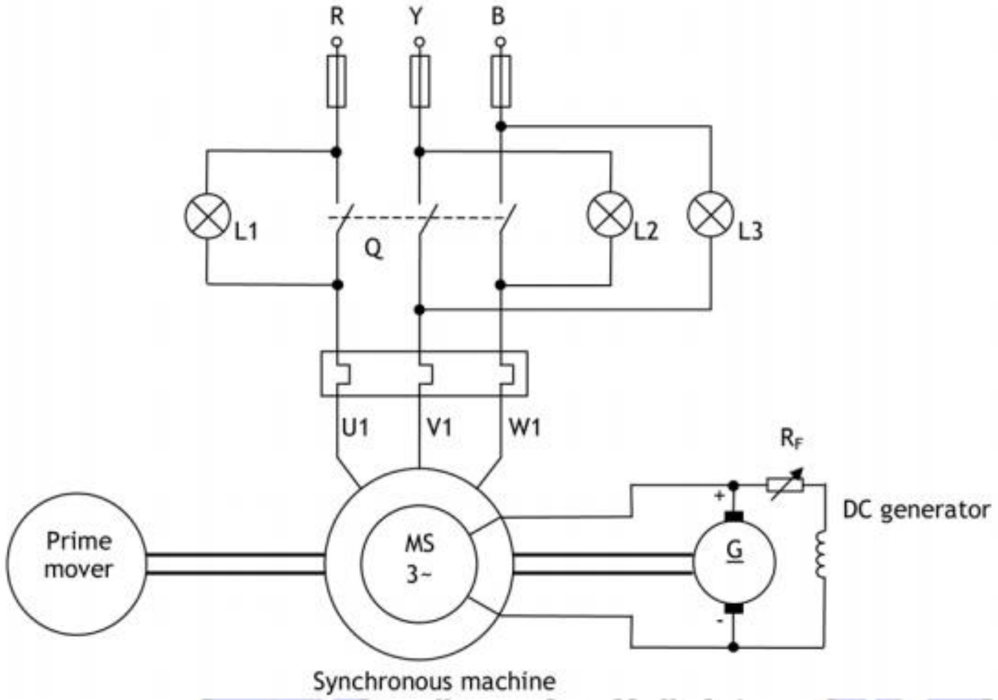
(iv) High-power electronic converters generating very low frequencies enable us to run synchronous motors at ultra-low speeds. Thus huge motors in the 10 MW range drive crushers, rotary kilns and variable-speed ball mills.

3-6 Methods of Starting:

1-Auxiliary drive: Driven unloaded by an auxiliary motor exc. voltage is applied while armature windings still open, motor operating as generator $P_{aux} = 10 - 20\% P_r$

2-Variable frequency starting $f = 0 \rightarrow f_r$

3-Induction start: Using squiral- cage winding called damper or starting winding. The circuit connection is shown. Reduce voltage starting method can be used E_x. winding should not be left open or shorted. When open a high emf is induced. When shorted a pulsating field producing positive torque T_1 and negative torque T_2 .When summing with the induction torque T_α given the resultant torque T_r. The pull-in torque(when applying dc voltage to the excitation winding) is not 95% n_s.



3-7 Stopping of Synchronous Motor:

Owing to the inertia rotor and its load, a large Syn. motor may take several hours to stop after being disconnect from the line to reduce the time we use the following braking methods:

1-Maintain full dc excitation with the armature in short circuit.

2-Maintain full dc excitation with the armature connect to three external resistors.

3-Apply mechanical braking.

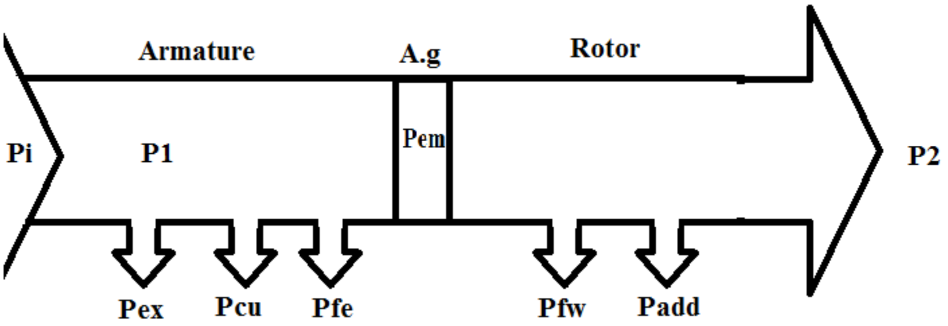
3-8 Motor Power:

$P_1 = \mathrm{P_i} - \mathrm{P_{ex}}$

$\mathrm{P_{em}} = \mathrm{P_1} - (P_{cu1} + P_{fe})$

$= \mathrm{P_{em}} - (\boldsymbol{P_{fw}} + \boldsymbol{P_{add}})$

$$T = \frac{P_2}{W_s} \quad , \quad T_m = \frac{V\,E_o}{W_s\,X_s} \quad , \quad P = \frac{3VE_o}{X_s} \times \sin\delta$$



Example1:

A 208 V, 45 KVA, 0.8 pf leading Δ connect 60 Hz syn. motor has a syn. reactance of 2.5Ω and negligible arm. resistance. Its core friction and winding losses are 2.5 Kw. The shaft initially supplying a 15-hp. load and the motor power factor is 0.8 leading. Find $\mathrm{I_a}$, $\mathrm{I_2}$,E_o , assume that the load increase to 30 hp, and find the pf.

Solution

$$\mathrm{P_2} = 15 \times 746 = 11190\,W$$

$$\mathrm{P_1} = \mathrm{P_2} + \mathrm{P_{losses}} = 11190 + 2500 = 13690\ \mathrm{W}$$

$$\mathrm{I_L} = \frac{P_1}{\sqrt{3}\ V\cos\theta} = \frac{13690}{\sqrt{3} \times 208 \times 0.8} = 47.5\ A$$

$$I_a = \frac{I_L}{\sqrt{3}} = \frac{47.5}{\sqrt{3}} = 27.4 \angle 36.87°\mathrm{A}$$

$$E_o = V - j\,I_a X_s = 208\angle° - j(27.4 \angle 36.87 \times (2.5)$$

$$= 249.1 - j54.8 = 255\angle - 12.4°\ \ voltage$$

After the load change:

$$P_1 = \mathrm{P_2} + \mathrm{P_{losses}} = 30 \times 764 + 2500 = 24880\,W$$

$$\delta = \sin^{-1}\frac{\mathrm{X_s\ P}}{\mathrm{3\ V\ E_o}} = \sin^{-1}\left[\frac{2.5\times24880}{2\times208\times255}\right] = 28°$$

$$\mathrm{I_a} = \frac{V - E_o}{\mathrm{jX_s}} = \frac{208\angle° - 255\angle - 23°}{\mathrm{j\ 2.5}} = 41.2\angle 15°$$

$$\mathrm{I_L} = \sqrt{3}\,\mathrm{I_a} = \sqrt{3} \times 41.2 = 71.4\ \mathrm{A} \qquad \cos\varphi = \cos 15 = 0.966\ \ Leading$$

Example 2:

Tow identical, 3-phase SG operating in parallel to supply load of 1MVA and 11kV, 0.8 lagging power factor. If the resistance and reactance of each generator are 5 and 50Ω respectively. If when varying the excitation of the first generator its current become 40A lag. Find: a) cosφ1 and cosφ2 b) I2.

Solution:

The load current is $I = \frac{P}{\sqrt{3}\,V\ \cos\varphi} = \frac{1X\ 10^6}{\sqrt{3}\ X 11 X 10^3 X\ 0.8} = 66A$

With active component of: $I_a = I\cos\varphi = 66x0.8 = 52.8$

And reactive component of: $I_a = I\sin\varphi = 66x0.6 = 39.6A$

$I_{a1}=I_{a2}=I_a/2 = 52.8/2= 26.4A,$

$I_{r1}=I_{r2}= I_r/2= 39.6/2=19.8A$

$I_1=I_2=I/2=66/2=33A$

When changing the excitation only reactive component is changing

$I_{r1}=\sqrt{I_1^2 - I_{a1}^2} = \sqrt{40^2 - 26.4^2} = 30A$

$I_{r2}= I_r - I_{r1} = 39.6-30=9.6A$

$I2= \sqrt{I_{a2}^2 + I_{r2}^2} = \sqrt{26.4^2 + 9.6^2} = 28A$

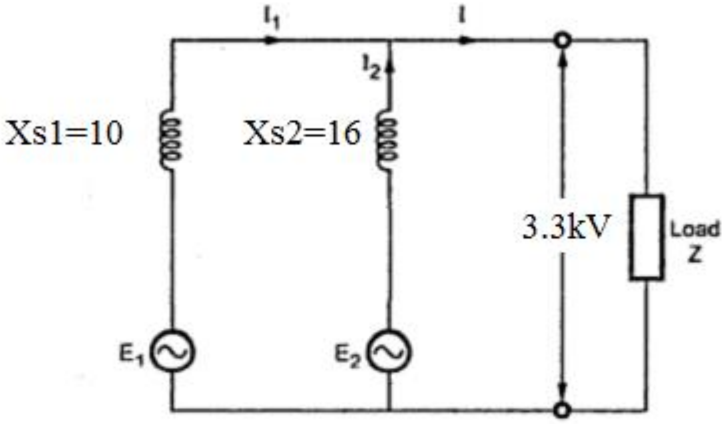
$$\cos\varphi 1 = \frac{I_{a1}}{I_1} = \frac{26.4}{40} = 0.66$$

$$\cos\varphi 2 = \frac{I_{a2}}{I_2} = \frac{26.4}{28} = 0.94$$

Example 3:

Two 3-phase, 3.3 kV, star connected alternators are connected in parallel to supply a load of 800kW at 0.8 power factor lagging. The prime movers are set that one machine delivers twice as much power as the other. The more heavily loaded machine has a synchronous reactance of 10Ω per phase and excitation is so adjusted that it operates at 0.75 lagging power factor. The synchronous reactance of the other machine is 16Ω per phase. Calculate the current, emf, power factor and load angle of each machine. The resistances may be neglected.

Solution:



The load apparent power:
S=P/cosφ

S= 800/0.8= 1000kVA

The load current is:

$I=\frac{S}{\sqrt{3}V}\ (\cos\varphi - J\sin\varphi)$

= 139.96-J104.97A

The current of first generator:

P_1=P 2/3= 800x2/3 = 533.33kW

$I_1=\frac{P_1}{\sqrt{3}V\ \cos\varphi}=\frac{533.33X10^3}{\sqrt{3}X\ 3.3X10^3X\ 0.75}=124.4<-14.4°$

Or I_1= 39.3-j82.3A

Then I_2= I-I_1 = 46.7-j22.7A

$E_1=V+jI_1X_{s1}=\frac{33..}{\sqrt{3}}+j10(39.3-j82.3)=2728+j933$

$=2888<18°V$

$E_2=V+jI_2X_{s2}=\frac{33..}{\sqrt{3}}+j16(46.7-j22.7)=2268+j747$

$=2388<18.2°V$

cosφ1=0.75   cosφ2= 0.899   load angle: δ1= 18º, δ =18.2º

Example 4:

A 3-phase, 3.3kV, Δ-connected, SG with 0.5Ω/phase effective resistance, supplying a 5.94MVA, Δ-connected 0.8 power factor load. If VR% is -5% find the alternator Xs,

Solution:

$$\frac{Eo - Vr}{Vr} = -0.05 \;\; then\; Eo = 0.95Vr = 3.135kV$$

$$I_r = \frac{MVA}{3Vr} = \frac{5.94\times10^6}{3\times3.3\times10^3} = 600A$$

IxR= 600x0.5 = 300V , IxX_s= 600X_s

$Eo = [(V\cos\varphi + IxR)^2 + (V\sin\varphi - IX_s)^2]^{1/2}$

$3.135^2 = (3.3x0.8+0.3)^2 + (3.3x0.6 + 0.6X_s)^2$

$X_s = 1.5\Omega$

4- Transformers

4-1 Introduction:

One of the main advantage of A.C transmission and distribution is the case with which an alternating voltage can be increase or reduce. The general practice in this country is to general at voltage of about 1-22 kV, then step up by means of transformers to higher voltages, for transmission lines, at suitable points other transformers are installed to step the voltage down to values suitable for motor, lamps heater, etc. Medium size transformers have a full-load efficiency of about 97-98 per cent, so that the losses at each point of transformation is very small. Also, since there are no moving parts, the amount of supervision required is practically negligible.

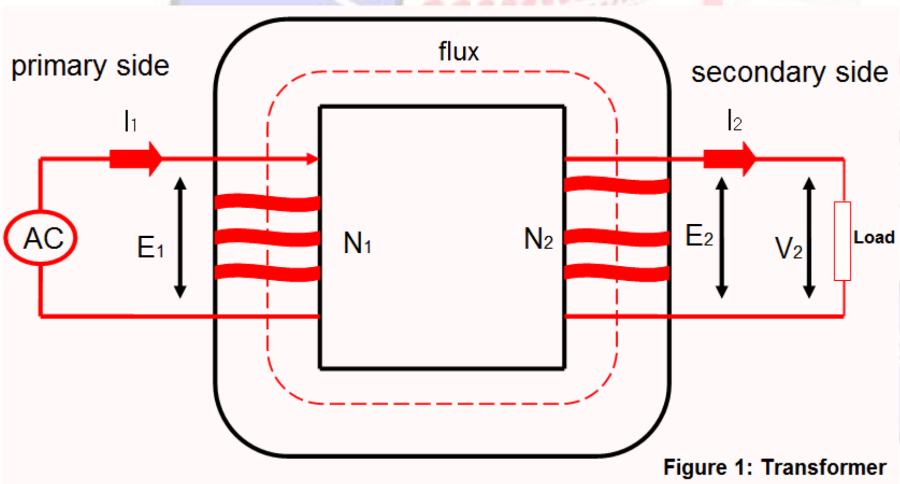
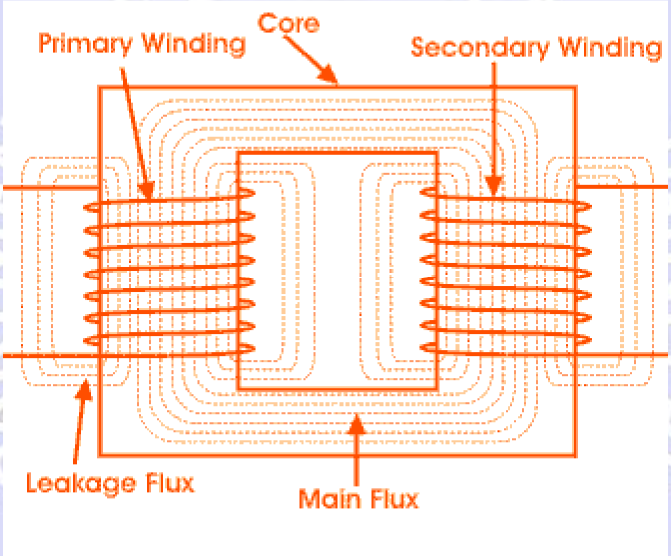


Figure 1: Transformer

4-2 Leakage Flux Responsible for The Induced Reactance of a Transformer:

In the preceding discussion, it has been assumed that all the flux linked with primary winding also links the secondary winding but, in practice it's impossible to realize this condition. It's found, however that all the flux linked with primary does it link the secondary but, part of it known $\emptyset_{L1}$ completes its magnetic cct. By passing air rather than around the core. The preceding section it was explain that the leakage flux is proportional to the primary and secondary current and that has effect is to induce e.m.f of self-induction in the winding. Consequently, the effect of leakage flux can be considered as equivalent to inductive reactors X_1 and X_2 connected in series with a transformer having no leakage flux.



4-2-1 Methods of reducing leakage flux:

The leakage flux can be practically eliminated by winding the primary and secondary, on over the other, uniformly around a laminate iron ring of uniform cross- section but, such an arrangement is not commercially practicable except is very small sizes. Owing to the cost of threading a large number of turns through the ring.

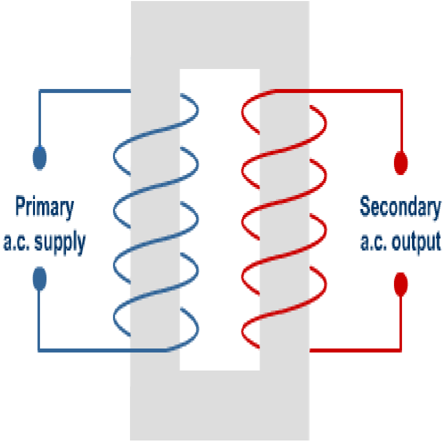
The principal methods used in practice are:

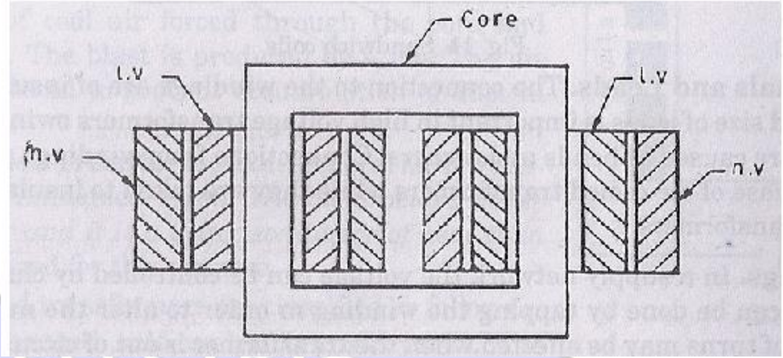
1-Making the transformer (window) long and narrow.

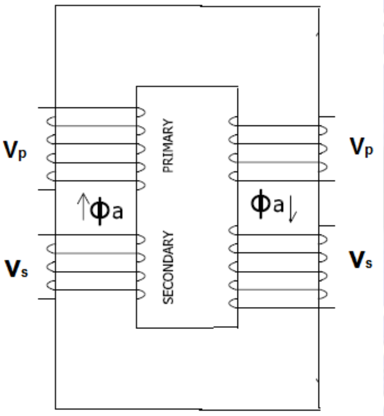
2-Arrangment the primary and secondary winding concentrically.

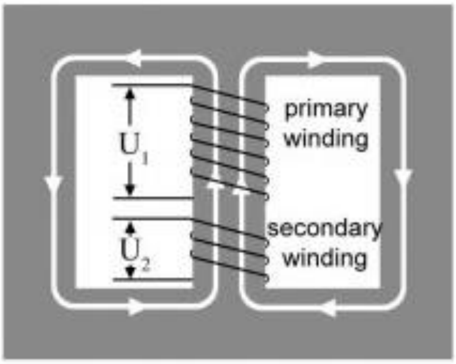
3-Sandwiching the primary and the secondary winding

4-using shell-type construction.









4-3 Principle of Action of a Transformer:

Fig (1) shown the general arrangement of transformer iron core (C) consist of laminated sheets, about 0.35mm thick, insulated from one another by thin layers paper or varnish or by spraying the laminations with a mixture of flour. The purpose of laminating the core is to reduce the loss due to eddy current induced by alternating magnetic flux. The primary coil is connected to supply and the secondary coil is connect to the load. An alternating voltage applied to primary circulated an alternating current through primary and this current produces an alternating flux in the iron core. The mean path of this flux being represented by the dotted D_1. If the whole of the flux produced by primary passes through secondary, the e.m.f induced in each urn is the same for P and S. Hence, if N_1 and N_2 be the number of turns on (P) and (S) respectively.

$$\frac{\boldsymbol{Total\ e.m.f\ induced\ in\ S}}{\textbf{Total e.m.f induced in P}} = \frac{\boldsymbol{N_2\ X\ e.m.f\ per\ turn}}{\boldsymbol{N_1\ X\ e.m.f\ per\ turn}} = \frac{\boldsymbol{N_2}}{\boldsymbol{N_1}}$$

When the secondary is on open circuit, its terminal voltage is the same as the induced e.m.f .The primary current is then very small, so that the applied voltage V_1 is practically equal and opposite to the induced in P hence:

$$\frac{\boldsymbol{V_2}}{\boldsymbol{V_1}} \approx \frac{\boldsymbol{N_2}}{\boldsymbol{N_1}}$$

Since the full-load efficiency of a transformers is, in early 100 per cent.

$I_a V_1$ X primary power factor $\approx I_2\ V_2$ X secondary power factor, but the primary and secondary power factor at full-load are nearly equal, $\frac{I_2}{I_1} = \frac{V_1}{V_2}$ And the we have:

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{V_1}{V_2}$$

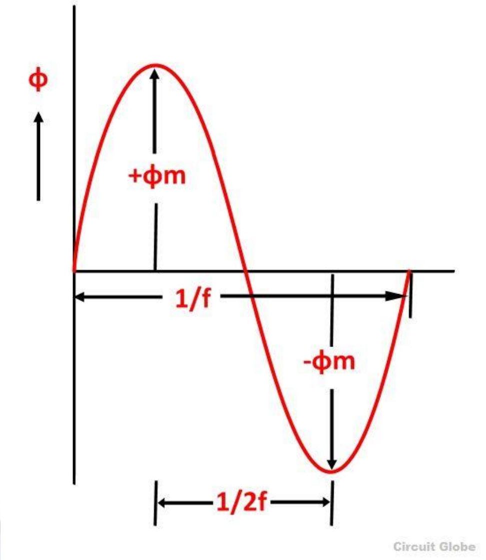
4-4 E.M.F Equation of a Transformer:

Suppose the maximum value of the flux to be $\emptyset_m$ Webbers and the frequency to be f hertz (or cycles /second).From fig(2) is seen that the flux has to change from $+\emptyset_m$ to $-\emptyset_m$ in half cycle , namely in $\frac{1}{2}$ F second. Average rate of change of

flux $=2\emptyset_m \div \frac{1}{2\,f}$

$=4f\emptyset_m$ weber/second……...

And average e.m.f induced turn $=4\ f\ \emptyset_m$ volts



For a sinusoidal wave the r.m.s or effective value is 1.11 times the average value. r.m.s value of e.m.f induced /turn =$1.11 \times 4\ f\ \emptyset_m$

Hence r.m.s value of e.m.f induced in primary =E_1.

$$E_1 = 4.44\ N_1\ f\ \emptyset_m\ \ volts$$

And r.m.s value of e.m.f induced in secondary = E_2

$$\mathrm{E}_2 = 4.44\ N_2\ f\ \emptyset_m\ \ \ volts$$

$$\emptyset_m\ = B_m \times A$$

B_m= maximum flux density

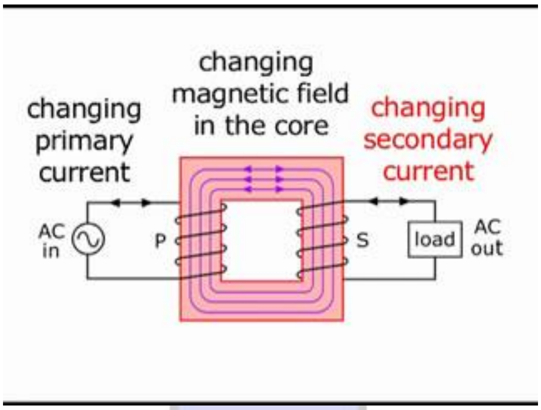
A=area of the core .

K=voltage transformation ratio

$$K=\frac{N_2}{N_1}$$

$$\frac{E_2}{E_1}=\frac{N_2}{N_1} = K$$

$$V_1 I_1 = V_2\ I_2 \rightarrow \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$



4-5 Windings the Ideal Transformer P And S:

R_1 and R_2 are resistance equal to the resistance of the primary and secondary windings of the actual transformer. Similarly, inductive reactance X_1 and X_2 represent the reactance of the windings due to leakage flux in the actual transformer. The inductive reactor X is that it takes a reactive current equal the magnetizing current I_{mag} of the actual transformer. The core losses due to hysteresis and eddy currents are allowed for by a resistor R of such value that it takes a current to equal to the core loss of the actual transformer.

4-6 Phasor Diagram for Transformer On no load:

It is most convenient to the commence the phasor diagram with the phasor representing the quantity that is common to the two windings, namely the flux $\emptyset$. This phasor can be convenient length and may be regarded merely as reference phasor, relative to which another phasor has to be drawn. The e.m.f induced by sinusoidal flux lags the flux by a quarter if a cycle. Consequently the e.m.f E_2 and E_1 induced in the secondary and primary winding are represented by phasor drawn 90° behind $\emptyset$, as in fig(3) .The values of E_2 and E_1 are proportion to the number of turns on the secondary and primary windings, since practically the whole of the flux set up by the primary is linked with the secondary when the latter is on open cct. For convenience in drawing phasor diagram for transformer, it will be assume that N_1 and N_2 are equal , so that $E_2 = E_1$, as shown in fig(3).Since the difference between the value of the applied voltage V_1 and that of the induced e.m.f

E_1 is only about 0.05 per cent when the transformer is on no load , the phasor representing V_1can be drawn equal and opposite to that representing E_1 .

The no-load current Io taken by the primary consist of two components: a reactive or magnetizing component (I_{mag}) producing the flux and therefore in phasor with the the latter, and an active or power component (I_c) supplying the hysteresis and eddy current losses in the iron core and the negligible (I_2R) loss in the primary winding. Components I_c in phase with the applied voltage ($I_c\ V_1$=core losses) this component is usually very small compared with I_{mag} , so that the no-load power factor is very small.

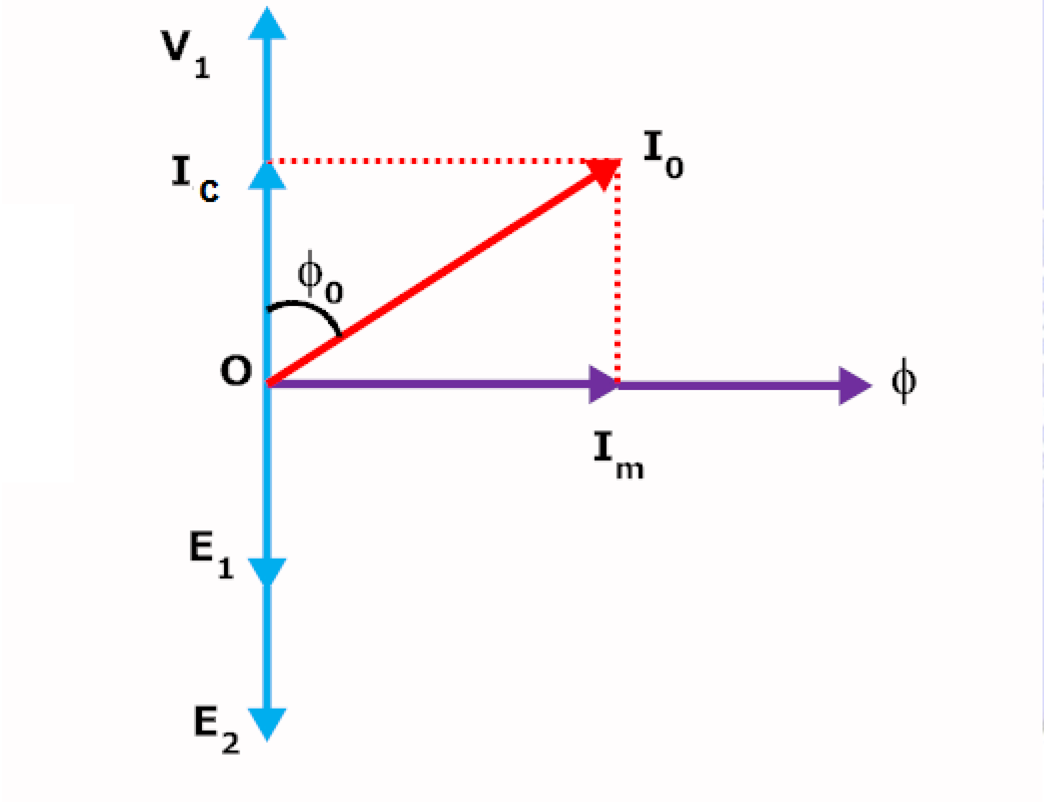


Figure 3 : Phasor diagram of practical transformer on no load

From (fig 3) it will see that:

No-load current =$I_o = \sqrt{i_c^2 + i_{mag}^2}$

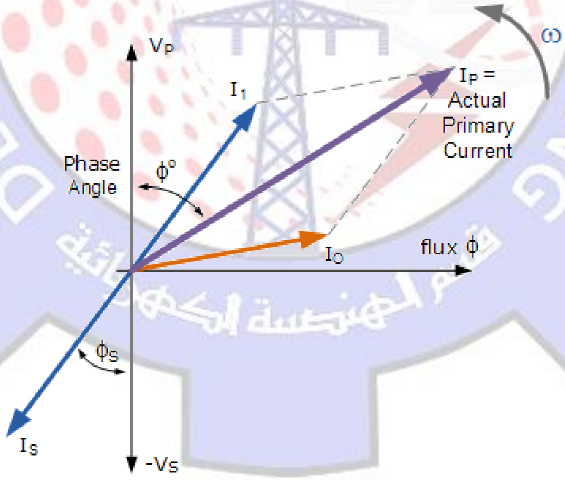
At power factor on no load=$\cos \emptyset_o = \frac{I_C}{I_O}$

$I_c = I_o \cos \emptyset_o$

$I_{mag} = I_o \sin \emptyset_o$

4-7 Phasor Diagram for A Loaded Transformer:

Assuming the voltage drop in the winding to be negligible with this assumption, it follows that the secondary voltage V_2 is the same as the e.m.f E_1 induced in the secondary, and the primary applied voltage V_1 is equal and opposite in phase to the e.m.f . E_1 induced in primary winding, also, if we again assume equal number of turns on the primary and secondary winding then $E_1 = E_2$. Let us consider the general case of a load having lagging power factor $\cos \emptyset_1$, hence the phasor representing the secondary current I_2 lags V_2 by angle $\emptyset_2$, as shown in fig (4), phasor I_1 represents the component of the primary current to neutralize the demagnetizing effect of the secondary current and is drawn equal and opposite to I_2 . Io is the no-load current of the transformer, the phasor sum of I_1 and is gives the total current I_1 taken from the supply, and the power factor on the primary side is $\cos \emptyset_1$, where $\emptyset_1$ is phase difference between V_1 and I_1 .



4-8 Phasor Diagram for A Transformer on Load:

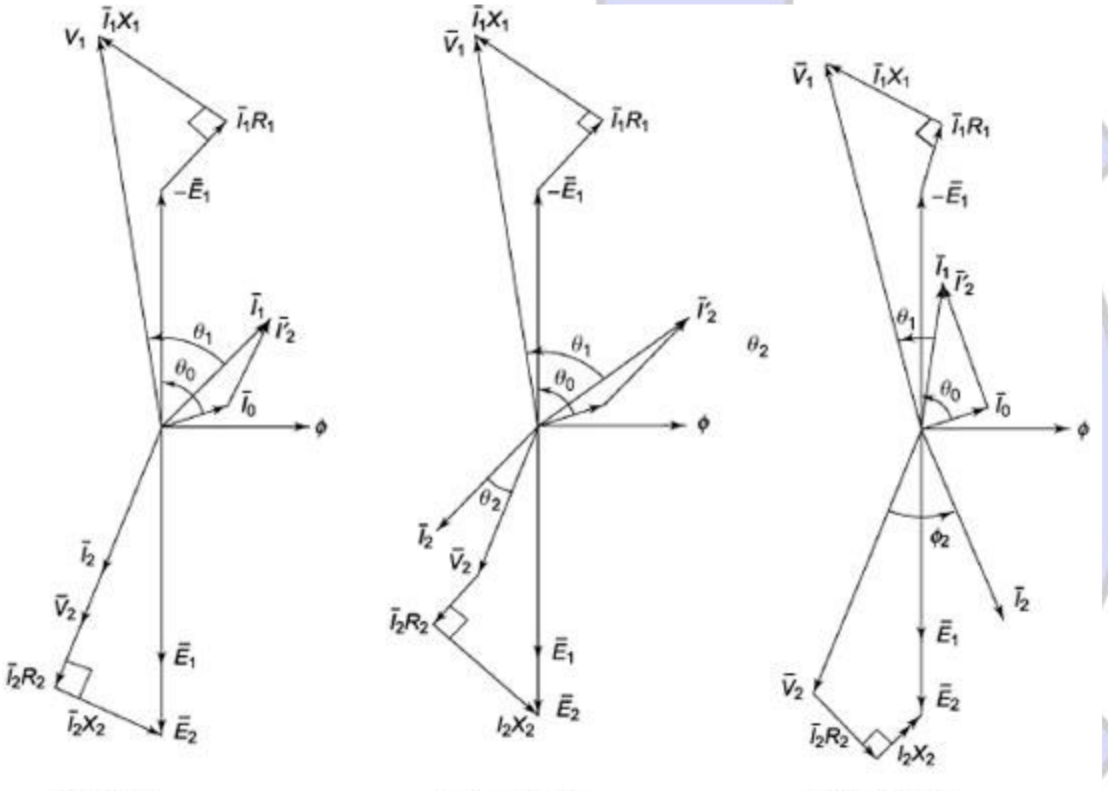
For convenience let us assume an equal number of turns on the primary and secondary windings, so that $E_1 = E_2$. Both E_1 and E_2 lag the flux by 90° as shown in fig (10) and $\grave{V}_1$ represents the voltage applied to the primary to neutralized the induced e.m.f E_1 , and is therefore equal and opposite to the latter. The general case of a load having a lagging P.f by 45° then

$I_1\ R_1$= Voltage drop due to primary resistance.

$I_2\ R_2$= Voltage drop due to secondary resistance.

$I_1\ X_1$= Voltage drop due to primary leakage reactance.

$I_2\ X_2$= Voltage drop due to secondary leakage reactance.



(a) Unity pf (b) Lagging pf (c) Leading pf

4-9 Equivalent Resistance:

A fig (5) is shown a transformer whose primary and secondary windings have resistance of R_1 and R_2 respectively. At would how be shown that the resistance of the two windings can be transferred to any on of the two windings. The advantage of concentrating both of the resistance in one winding is that it makes calculations very simple and easy because on has then two works in one winding only. As will be proved that the resistance of R2 in secondary is equivalent to $\frac{R_2}{K^2}$ in primary. The value $\frac{R_2}{K^2}$ will be denoted by $\grave{R}_2$ – the equivalent secondary resistance referred primary. The copper losses in secondary is $I_2^2\ R_2$. This loss is supplied by primary which takes a current of I_1 if $\grave{R}_2$ is the equivalent resistance in primary which would are caused the same loss as R_2 in secondary, then:

$I_1^2 \grave{R}_2 = I_2^2 R_2$ or $\grave{R}_2 = (\frac{I_2}{I_1})^2\ R_2$

$\frac{I_2}{I_1} = \frac{1}{K} \rightarrow \grave{R}_2 = \frac{R_2}{K^2}$

And $I_1^2\ R_1 = I_2^2\quad Or\quad \grave{R}_1 = (\frac{I_1}{I_2})^2\ R_1$

$\frac{I_1}{I_2} = K \rightarrow \grave{R}_1 = K^2\ R_1$

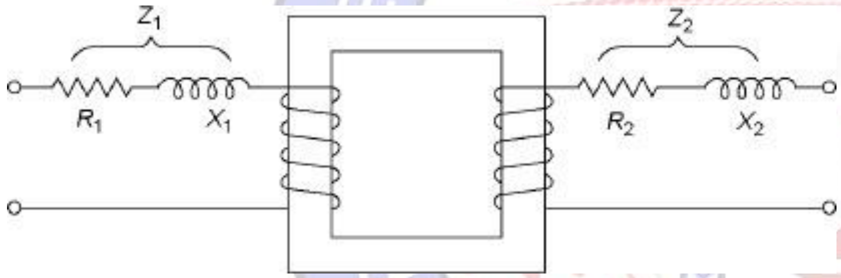


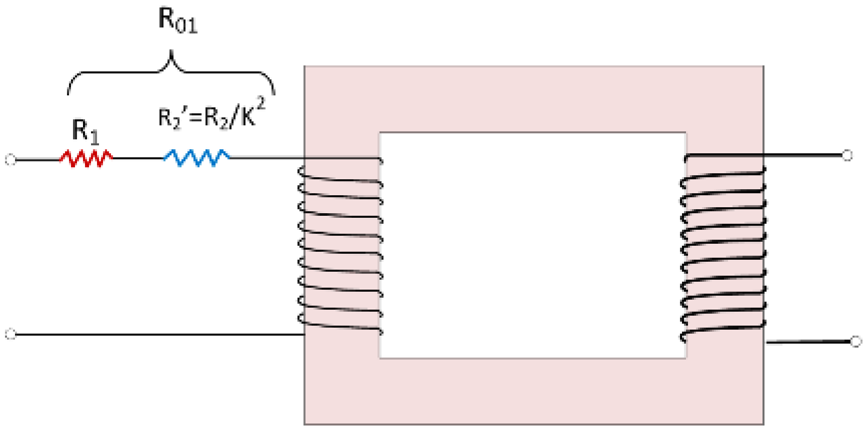
Fig (6, a), secondary resistance has been transferred to primary side leaving secondary circuit resistance less. Resistance $R_1 + \grave{R}_2 = R_1 + \frac{R_2}{K^2}$ is known as equivalent or effective resistance of the transformer as referred to primary and may be designated as R_{01}:

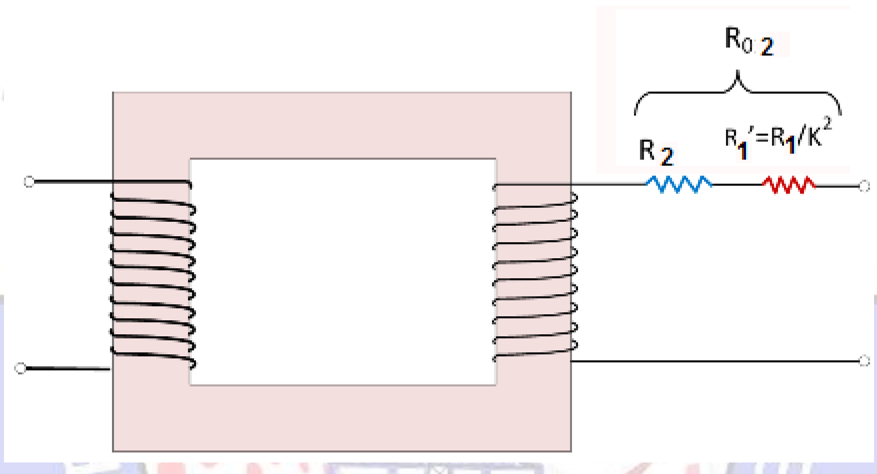
$R_{o1} = R_1 + \grave{R}_2 = R_1 + \frac{R_2}{K^2}$

Similarly, the equivalent resistance of the transformer as referred to secondary is:

$R_{o2} = R_2 + \grave{R}_1 = R_2 + R_1\ K^2$

As shown in fig (6, b)





4-10 Equivalent Circuit of A Transformer:

The behavior of a transformer may be conveniently considered by assuming it to be equivalent to an ideal transformer, a transformer having no losses and no magnetic leakage and an from core of infinite permeability requiring no magnetizing current, and then allowing for the imperfections of the actual transformer by means of additional circuits or impedance inserted between the supply and the primary winding and between the secondary and the load, thus in fig(9).

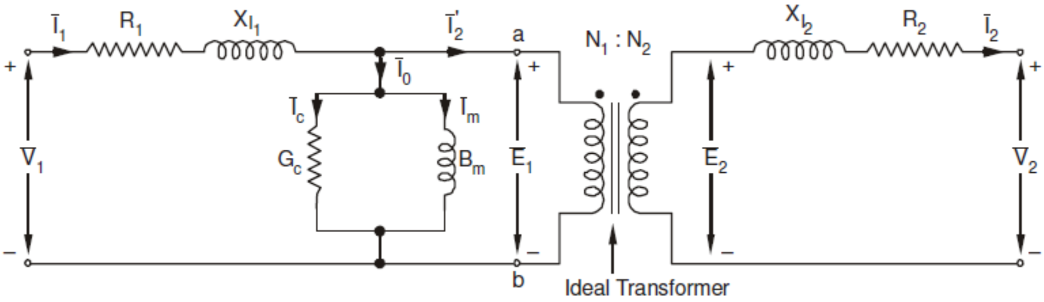
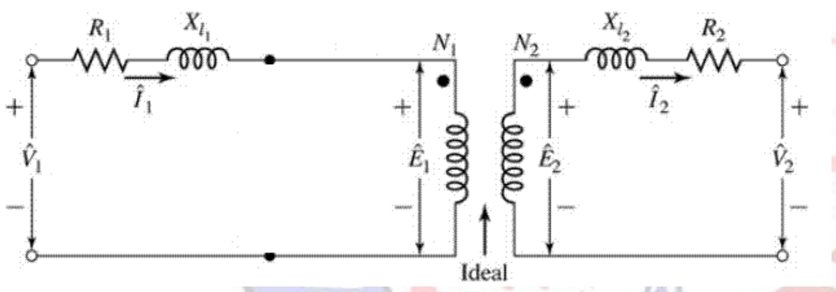


Fig. (9) Equivalent circuit of a transformer

4-11 Approximate Equivalent cct. of Transformer:

Since the no-load of transformer is only about 3.5 per cent of the full-load primary current, we can omit the parallel circuits R and X in fig (9) without introducing an appreciable error when we are considering the behavior of the transformer on full-load. Thus, we have the equivalent cct. Of fig (11)



Simplification of The Approximate Equivalent Circuit Of A Transformer:

We can replace the resistance R_2 of the secondary of fig(11) by additional resistance $\grave{R}_2$ in the primary circuit such the power absorbed in $\grave{R}_2$ when carrying the primary current is equal to that in R_2 due to the secondary current.

$$I_1^2\ \grave{R}_2 = I_2^2\ R_2 \rightarrow\ \grave{R}_2 = (\frac{I_2}{I_1})^2 \approx\ R_2(\frac{V_1}{V_2})^2$$

Hence if R_e be a single resistance in the primary circuit equivalent. The primary and secondary resistances of the actual transformer.

$$R_e = R_1 + \grave{R}_2 = R_1 + R_2\ (\frac{V_1}{V_2})^2$$

Similarly, since the inductance of a coil is proportional to the square of the number of turns , the secondary leakage reactance X_2 can be replace by an equivalent reactance $\grave{X}_2$ in the primary circuit , such that:

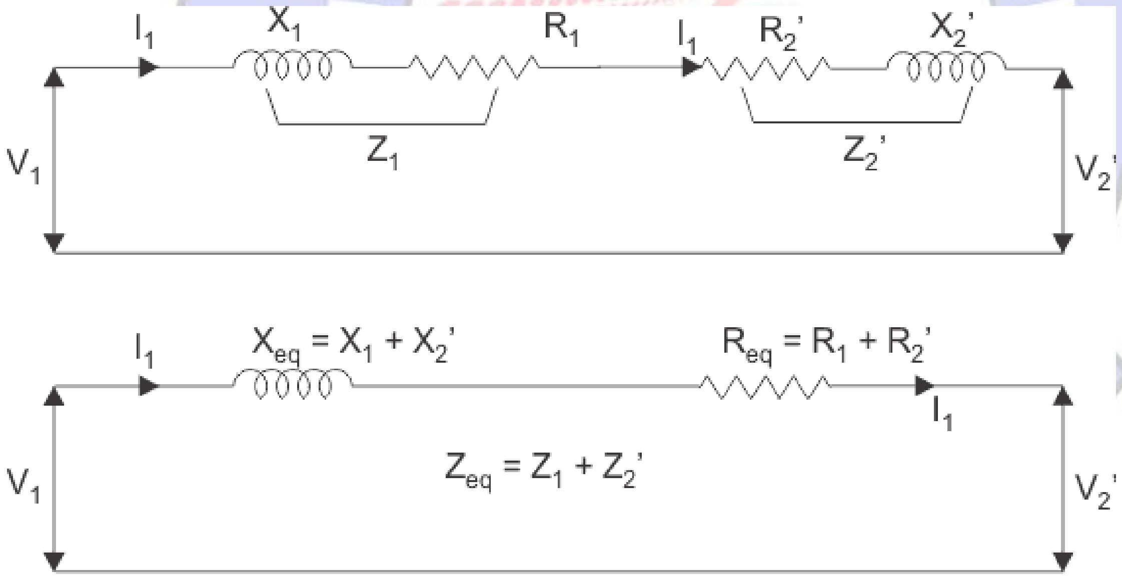
$$\grave{X}_2 = X_1 \left(\frac{N_1}{N_2}\right)^2 \approx X_1 (\frac{V_1}{V_2})^2$$

X$_e$ be the single reactance in the primary cct . equivalent X$_1$ and X$_2$ of the actual transformer.

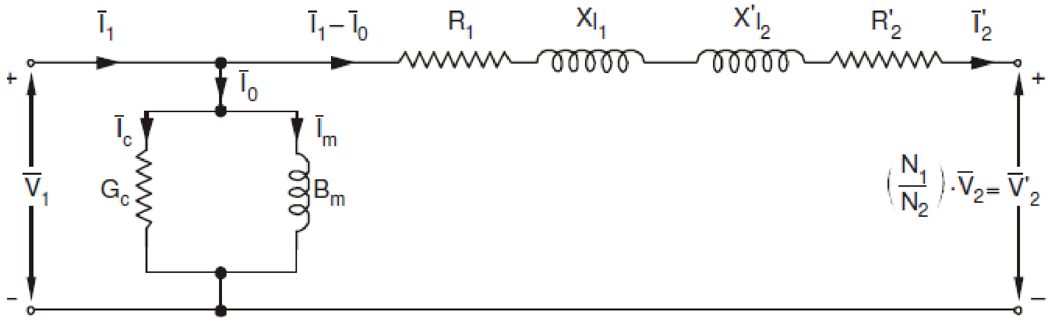
$$X_e = X_1 + \grave{X}_2 = X_1 + X_2 \left(\frac{V_1}{V_2}\right)^2$$

Z_e be the equivalent impedance of the primary and secondary winding referred to primary cct.

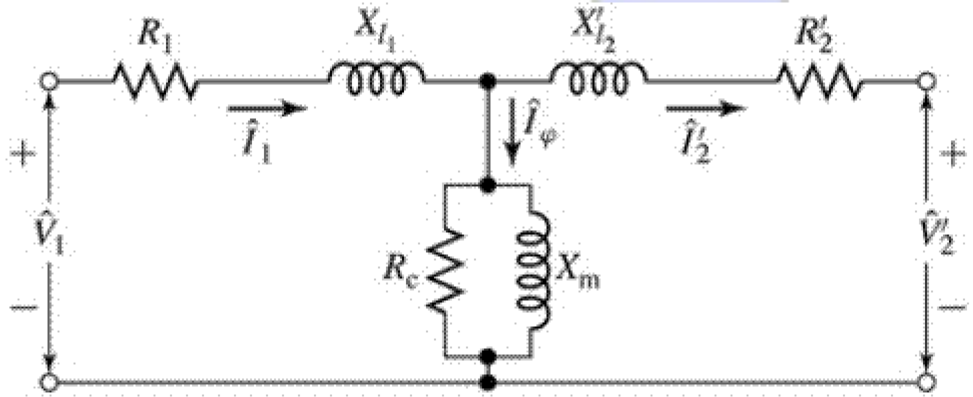
$$Z_e = \sqrt{R_e^2 + X_e^2} \quad , R_e = Z_e \cos\varphi \, , \quad X_e = Z_e \sin\varphi$$



Approximate Equivalent Circuit of Transformer referred to Primary



(a) Equivalent Circuit Referred to Primary Side



4-12 Efficiency of A Transformer:

The losses which occur in a transformer on load can be divided into two groups:

1-Copper losses in primary and secondary windings namely $I_1^2\, R_1 + I_2^2\, R_2$

2-Iron losses in the core due to hysteresis and eddy currents. The factor determining these losses have already been discussed in A.C machines.

Since the maximum value of the flux in a normal transformer does not vary by more than about 2 per cent between no load and full load its usual to assume the iron losses constant at all loads.

$P_c = total\ iron\ loss\ in\ core$

$$\sum_{loss} = P_c + P_{cu1} + P_{cu2} = P_c + I_1^2 R_1 + I_2^2\, R_2$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + losses} = \frac{I_2 V_2 \times Pf}{I_2 V_2 \times Pf + P_c + I_1^2 R_1 + I_2^2 R_2}$$

OR $\frac{P_{in}-losses}{P_{in}} = 1 - \frac{losses}{P_{in}}$

$$P_c = V_1 I_o \cos \emptyset o$$

$$P_{cu} = P_{cu1} + P_{cu2} = I_1^2 R_1 + I_2^2 R_2$$

4-13 Condition for Maximum Efficiency of Transformer:

R2e the equivalent resistance of the primary and secondary windings referred to the secondary circuit.

$$R_{2e} = R_1 (\frac{N_2}{N_1})^2 + R_2$$

=a constant for gainer transformer

For any load current I_2,

Total copper loss= $I_2^2 R_{2e}$

And ŋ%= $\frac{I_2 V_2 \times P.f}{I_2 V_2 \times P.f + P_c + P_{cu2}} = \frac{I_2 V_2 \times P.f}{I_2 V_2 \times P.f + P_C + I_2^2 R_{2e}}$

For a normal transformer, V_2 is approximately constant, hence for a load of given power factor. The efficiency is maximum when the denominator of ŋ is a minimum.

$$\frac{d}{dI_2}(V_2 I_2 \times P.f + P_c + I_2^2 R_{2e}) = 0$$

$$\frac{d}{dI_2}\left(V_2 \times P.f + \frac{P_c}{I_2} + I_2 R_{2e}\right) = 0$$

$$\frac{-P_c}{I_2^2} + R_{2e} = 0 \rightarrow I_2^2 R_{2e} = P_c$$

Example1:

A 200 KVA, 6600V /400V, 50 HZ single phase transformer has 80 turns on the secondary. Calculate:

a- The approximate values of the primary and secondary current.
b- The approximate number of primary turns.

c- The maximum value of the flux.

Solution:

a- full-load primary current $I_1 = \frac{S}{V_1} = \frac{200 \times 10^3}{6600} = 30.3\ A$

- full-load secondary current $I_2 = \frac{S}{V_2} = \frac{20 \times 10^3}{400} = 500\ A$

b- $N_1 = N_2 \times \frac{V_1}{V_2} = \frac{80 \times 6600}{400} = 1320$

c- $E_2 = 4.44 \times N_2 \times f \times \emptyset_m$

$400 = 4.44 \times 80 \times 50 \times \emptyset_m$

$\emptyset_m = 0.0225\ wb$.

Example 2: A 2200/200 V transformer draws a no-load primary current of 0.6 A and absorb 400 watts, find the magnetic and iron loss current.

Solution: iron loss current $= I_c = \frac{P}{V} = \frac{400}{2200} = 0.182\ A$

$I_o = \sqrt{I_o^2 + I_{mag}^2} \rightarrow I_o^2 = I_c^2 + I_{mag}^2$

$0.6^2 = 0.182^2 + \mathrm{I}_{\mathrm{mag}}^2$

$I_{\mathrm{mag}} = 0.572$ A

Example 3: A single phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current 3A at a power factor 0.2 lagging. Calculate the primary current and power factor when the secondary current is 280 A, power factor of 0.8 lagging. (voltage drop-in winding be negligible).

Solution:

If $\grave{I}_1$ represents the component of the primary current to neutralize the demagnetizing effect of the secondary current. The power –turns due to $\grave{I}_1$ must be equal and opposite to those due to I_2.

$$\frac{I_2}{\grave{I}_1} = \frac{N_1}{N_2} = \frac{280}{\grave{I}_1} = \frac{1000}{200}$$

$$\grave{I}_1 \times 1000 = 280 \times 200 \rightarrow \grave{I}_1 = 56\ A$$

$$\cos\emptyset_2 = 0.8 \rightarrow \sin\emptyset_2 = 0.6$$

$$\cos\emptyset_o = 0.2 \rightarrow \sin\emptyset_o = 0.98$$

$$I_1\cos\emptyset_1 = \grave{I}_1\cos\emptyset_2 + I_o\cos\emptyset_o$$

$$= 56 \times 0.8 + 3 \times 0.2 = 45.4\ A$$

$$I_1\sin\emptyset_1 = \grave{I}_1\sin\emptyset_2 + I_o\sin\emptyset_o$$

$$= 56 \times 0.6 + 3 \times 0.98 = 36.54\ A$$

$$I_1 = \sqrt{45.4^2 + 36.54^2} = 58.3\ A$$

$$\tan^{-1}\frac{I_1\sin\emptyset_2}{I_1\cos\emptyset_1} = \tan^{-1}\frac{36.54}{45.4} = 38°$$

Primary power factor= $\cos\emptyset_1 = \cos 38° = 0.78\ lagging$

Example 4:

A 50 KVA, 4400/220 V transformer has R_1=3.45Ω, R_2=0.009 have values of reactance are X_1=5.2Ω and X_2=0.015Ω. Calculate for the transformer:1-equivalent resistance as referred to primary. 2-equivalent resistance as referred to secondary. 3-equivalent reactance as referred to both primary and secondary. 4-equivalent impedance as referred to both primary and secondary. 5-total P_{cu} loss. first used individual resistance two windings and secondary using equivalent resistance as referred to each side.

Solution: $I_1 = \frac{S}{V_1} = \frac{50\times10^3}{4400} = 11.36\,A$

$I_2 = \frac{S}{V_1} = \frac{50\times10^3}{220} = 227\,A$

$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{220}{4400} = 0.05$

$\mathrm{R_{e1}} = \mathrm{R_1} + \frac{\mathrm{R_2}}{\mathrm{K^2}} = 3.45 + \frac{0.009}{0.05^2} = 7.05\Omega$

$R_{e2} = K^2 R_{e1} = 0.05^2 \times 7.05 = 0.0176\Omega$

Or $R_{e2} = R_2 + \frac{R_1}{K^2} = 0.009 + \frac{3.45}{0.05^2} = 0.0176\,A$

$X_{e1} = \mathrm{X_1} + \frac{X_2}{\mathrm{K^2}} = 5.2 + \frac{0.015}{0.05^2} = 11.2\Omega$

$X_{e2} = \mathrm{X_2} + \frac{\mathrm{X_1}}{\mathrm{K^2}} = 0.015 + \frac{5.2}{0.05^2}$

$X_{e1} = k^2 X_{e1} = 0.05^2 \times 11.2 = 0.028\Omega$

4- $Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{7.05^2 + 11.2^2} = 13.23\Omega$

$Z_{e2} = \sqrt{R_{e2}^2 + X_{e2}^2} = \sqrt{0.0176^2 + 0.028^2} = 0.03311\Omega$

5-$P_{cu} = I_1^2 R_1 + I_2^2 R_2 = 11.36^2 \times 3.45 + 227^2 \times 0.009 = 910\mathrm{W}$

$\mathrm{P_{cu}} = \mathrm{I_1^2 R_{e1}} = 11.36^2 \times 7.05 = 910\,W$

$= I_2^2 R_{e2} = 227^2 \times 0.017 = 910\,w$

Example 5: The primary and secondary of a 500 KVA transformer have resistance of 0.42Ω and 0.0011Ω respectively. The primary and secondary voltages are 6600 V and 400 V, and the iron losses is 2.9 kW. Calculate the efficiency at:

a-full-load, b-half-load. Assuming the p.f of the load 0.8.

solution: $I_2 = \frac{S}{V_2} = \frac{500\times10^3}{400} = 1250\ A$

$I_1 = \frac{S}{V_1} = \frac{500\times10^3}{6600} = 75.8\ A$

$P_{cu2} = \mathrm{I}_2^2\mathrm{R}_2 = 1250^2 \times 0.0011 = 1720\ W$

$P_{cu1} = I_1^2 R_1 = 75.8^2 \times 0.42 = 2415\ W$

$\mathrm{P_{cu}} = \mathrm{P_{cu1}} + \mathrm{P_{cu2}} = 4135 + 2.9 \times 10^3 = 7035\ \mathrm{W}$

$\mathrm{P_2} = \mathrm{P_{out}} = \mathrm{S_x} \cos\varphi = 500 \times 10^3 \times 0.8 = 400\ \mathrm{KW}$

$\mathrm{P_1} = \mathrm{P_{in}} = \mathrm{P_2} + \sum P_{loss} = 400 + 7.035 = 407.035\ \mathrm{KW}$

$\frac{400}{407..035} \times 100 = 98.27\%$η=

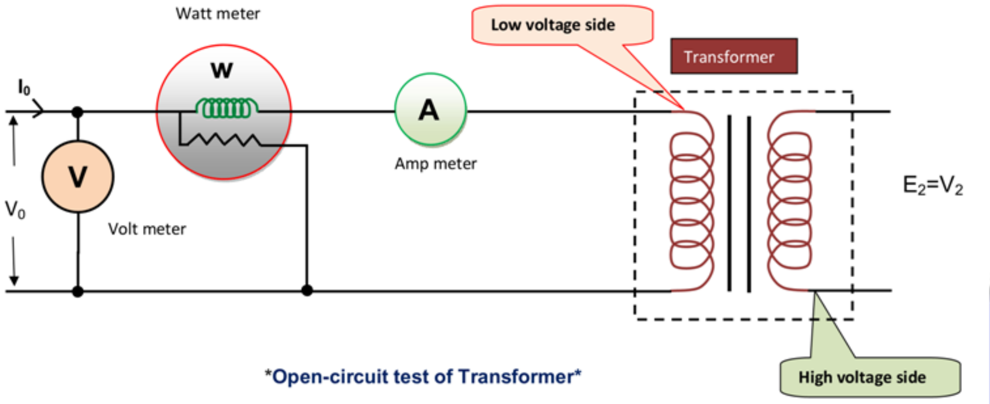
b-half load $P_{cu} = 4135 \times 0.5^2 = 1034\ W$

$\sum P_{loss} = 1034 + 2.9 = 3.934\ KW$

$\frac{200}{203.9} \times 100 = 98.08\%$η=

4-14 Open Circuit or No-Load Test:

The purpose of this test is to determine no-load loss or core loss and no-load current I_o which is helpful in finding $X_o and\ R_o$.



Open-circuit test of Transformer

$$P(w) = I_o V_1 \cos \emptyset_o$$

$$I_{mag} = I_o \sin \emptyset_o$$

$$\mathrm{I_c} = \mathrm{I_o} \cos \emptyset_o$$

$$\mathrm{X_o} = \frac{V_1}{\mathrm{I_{mag}}} \quad , \ \mathrm{R_o} = \frac{\mathrm{V_1}}{\mathrm{I_c}}$$

Wher: $X_o = no\ load\ inductance, R_o = no\ load\ resistance$

$I_o = no\ load\ current\ ,\ V_1 = supply\ voltage$

$P = no\ load\ power$

If Wor P_o is wattmeter reading, then:

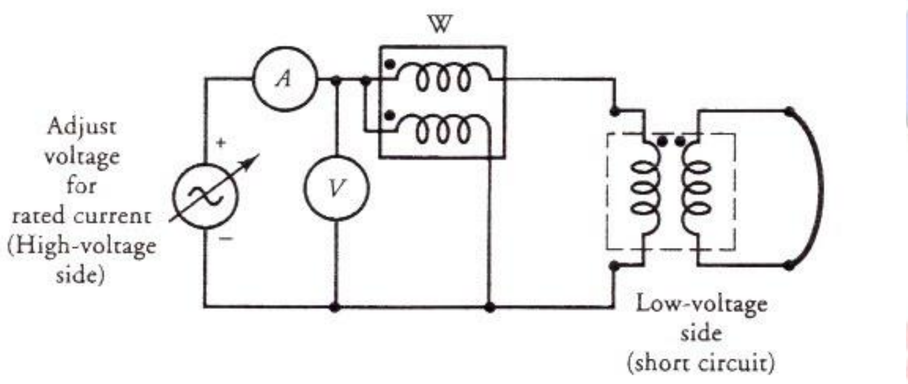
$P_o = V_1 I_o \cos \emptyset_o$ and $\cos \emptyset_o = \frac{\mathrm{P_o}}{\mathrm{V_1 I_o}}$

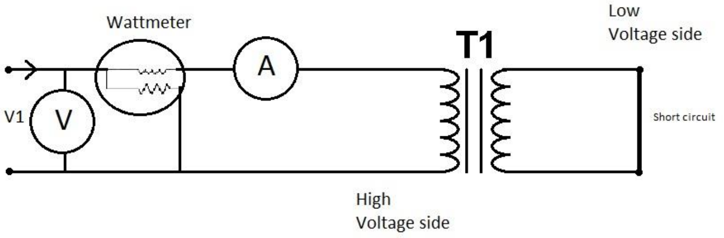
4-15 Short Circuit or Impedance Test:

This is an economical method for determining the follow:

$Z_{e1}, Z_{e2}, X_{e1}, X_{e2}, P_{cu}$. Equivalent impedance ($Z_{e1} or\ Z_{e2}$), leakage reactance ($X_{e1}\ or\ X_{e2}$) and total resistance ($R_{e1}\ or\ R_{e2}$) of the transformer as referred to the winding.

P_{cu} losses at full load and at any load and then used to calculate the efficiency of transformer. Knowing $Z_{e1}\ and\ Z_{e2}$, the total voltage drops in the transformer as referred to primary or secondary can be calculate and hence regulation of the transformer determined.





If Vsc is the voltage required to circulate rated load current , then:

$$Z_{e1} = \frac{V_{sc}}{I_1} \quad , \quad X_{eq} = \frac{V_1}{I_1}$$

ALSO

$P_{sc} = \mathrm{I}_1^2\ \mathrm{R}_{e1}$

$\mathrm{R}_{e1} = \frac{P_{sc}}{\mathrm{I}_1^2}$ and $\mathrm{X}_{e1} = \sqrt{\mathrm{Z}_{e1}^2 - \mathrm{R}_{e1}^2}$

Note:

P_o =imput power in watts on the open cct test

= Iron loss =P_c

P_{sc} = imput power in watts on the short cct test with full load current.

P_{sc} = total copper loss on full load,

$\sum loss\ in\ full\ load = P_o + P_{sc}$

$\frac{full\ load\ VA \times P.f}{full\ load\ VA \times P.f + P_o + P_{sc}} \times 100$ŋ=

4-16 Voltage Drop in A Transformer:

Total transformer drops as referred to secondary

$= I_2\ R_{e1}\ \cos\varphi \mp \mathrm{I}_2\ \mathrm{X}_{e2}\ \sin\varphi$

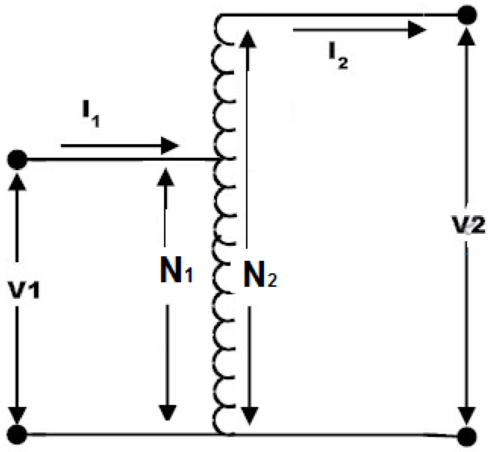
Total transformer drops as referred to primary

$= I_1\ R_{e1}\ \cos\varphi \mp I_1\ X_{e1}\ \sin\varphi$

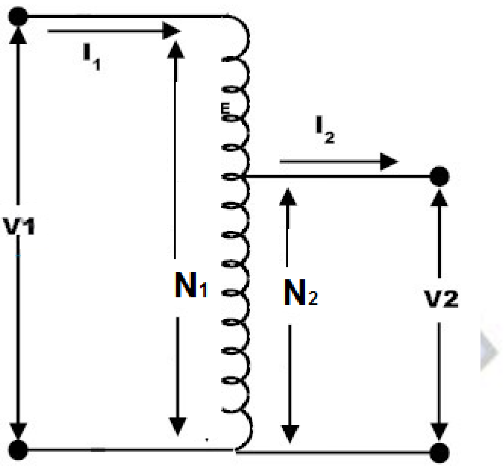
Note: the upper sings are to be used +Ve for lagging power factor and – Ve for leading power factor.

4-17 Auto – transformer:

It is transformer with one winding only. Because of on winding it use less cupper and hence is cheaper. Figure below shows both step down and step up auto-transformer.



(a)Step up Autotransformer



(b)Step down Autotransformer

As compared to an ordinary 2-winding transformer of same output, an auto-transformer has higher efficiency but ,smaller size.

Uses of auto-transformer:

1-To give small boost to a distribution cable to current the voltages drop.

2-To give up to 50 to 60   at full voltage to an induction motor during starting.

3- As furnace transformers for getting a convenient supply to suit the furnace winding from 230 v supply.

4- As interconnecting transformer in 132kv/330kv system.

5- In control equipment for 1-phase and 3-phase electrical locomotives.

5 Parallel operation of single-phase transformer:

For supply a load excess of the rating of an existing transformer, a second transformer may the connected in parallel. There are certain definite conditions which must be satisfied in order to avoid any local circulating currents and to ensure that the transformers share the common load in proportion to their KVA ratings. The conditions are:

1-Primary windings of the transformer should be suitable for the supply system voltage and frequency.

2-The transformers should be properly connected with regard to polarity.

3- The voltage ratings of both primaries and secondaries should be identical.

4-The percentage impedance should be equal in magnitude and have the same X/R ratio in order to avoid circulating currents and operation at different power factor.

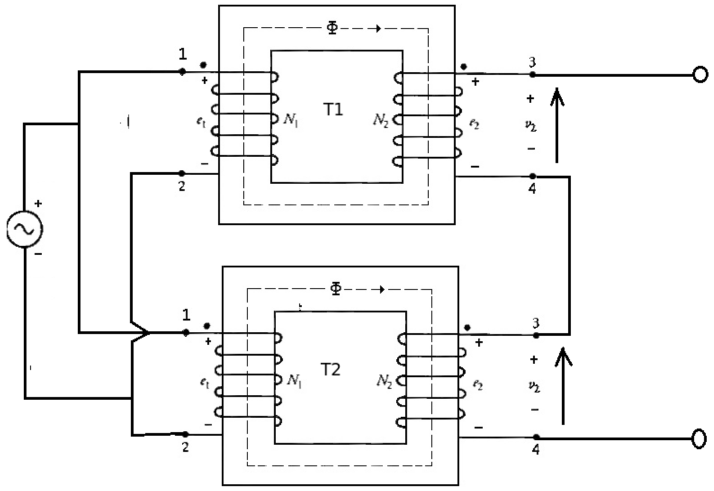
5- With transformers having different KVA ratings. The equivalent impedances should be inversely proportional to the individual KVA rating if circulating currents are to be avoided.

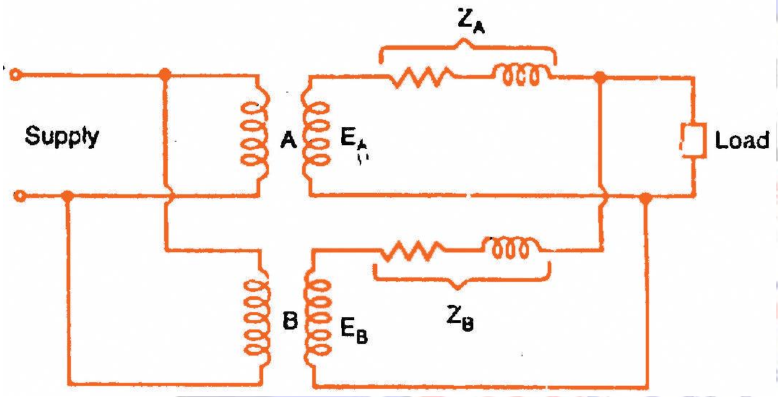
There are two cases for represent of parallel connections of single-phase transformers:

5-1 Ideal case:

Equal voltage ratios:

At no-load of both secondaries is the same $E_A=E_B=E$, and that the two voltage are coincident . There is no phase different between E_A and E_B which would be turn if the magnetizing different from each other.





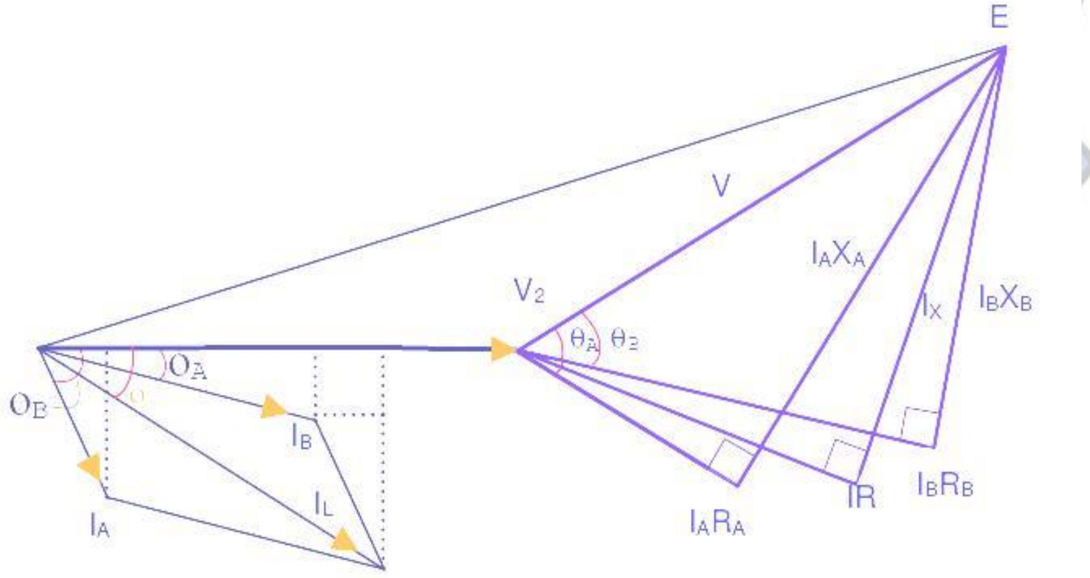


Figure 39: Phasor Diagram of Operation for two Transformers working in Parallel

Phasor diagram of 2 transformers in parallel:

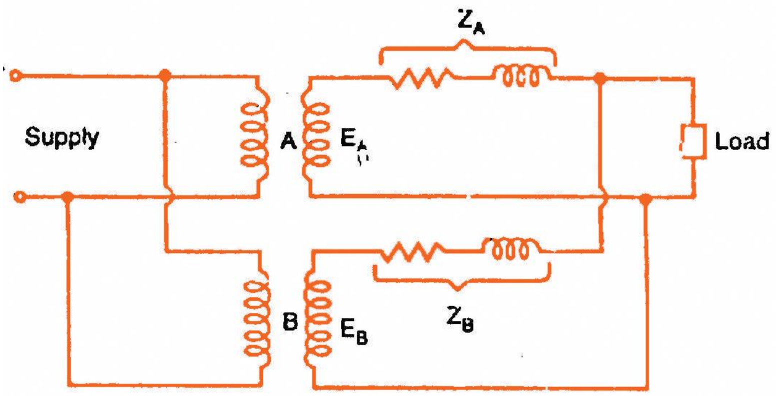
$$\mathrm{I_A} = \frac{\mathrm{I\,Z_B}}{Z_A + \mathrm{Z_B}} \quad and \quad I_B = \frac{I\,Z_A}{Z_A + Z_B}$$

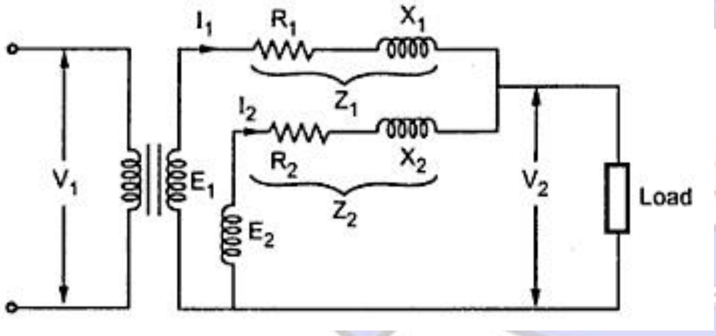
$$\mathrm{V_2 I_A} = \frac{V_2\,\mathrm{I\,Z_B}}{\mathrm{Z_A + Z_B}} \quad and \quad V_2 I_B = \frac{V_2\,I\,Z_A}{Z_A + Z_B}$$

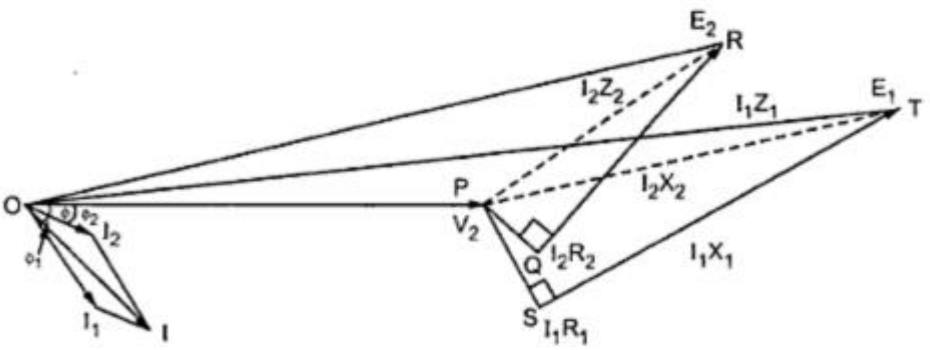
$$\mathrm{S_A} = S\,\frac{Z_B}{Z_A + Z_B} \quad and \quad S_B = S\,\frac{Z_A}{Z_A + Z_B}$$

Unequal Voltage Ratios:

In this case the voltage ratios of the two transformers are different. It means that their no-load secondary voltages are unequal.







$$E_A = \mathrm{I_A\,Z_A + V_2}$$

$$E_B = I_B\, Z_B + V_2$$

$$V_2 = I\, Z_L = (I_A + I_B) Z_L$$

$$E_A = I_A\, Z_A + (I_A + I_B) Z_L$$

$$E_B = I_B\, Z_B + (I_A + I_B) Z_L$$

$$E_A - E_B = I_A\, Z_A - I_B\, Z_B$$

$$\mathrm{I_A} = \frac{E_A - E_B) + I_B\, Z_B}{Z_A} \qquad Substituting\ I_A\ in$$

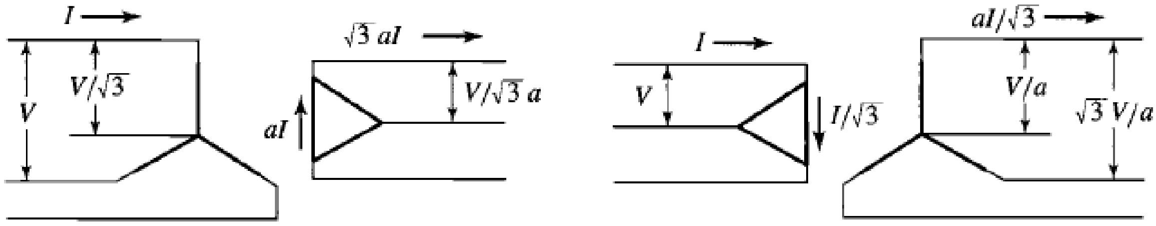
$$\mathrm{I_B} = \frac{E_B\, \mathrm{Z_A} - (\mathrm{E_A} - \mathrm{E_B})\mathrm{Z_L}}{\mathrm{Z_A\, Z_B} + \mathrm{Z_L}(\mathrm{Z_A} + \mathrm{Z_B})}$$

$$\mathrm{I}_A = \frac{E_A\, \mathrm{Z_B} - (\mathrm{E_A} - \mathrm{E_B})\mathrm{Z_L}}{\mathrm{Z_A\, Z_B} + \mathrm{Z_L}(\mathrm{Z_A} + \mathrm{Z_B})}$$

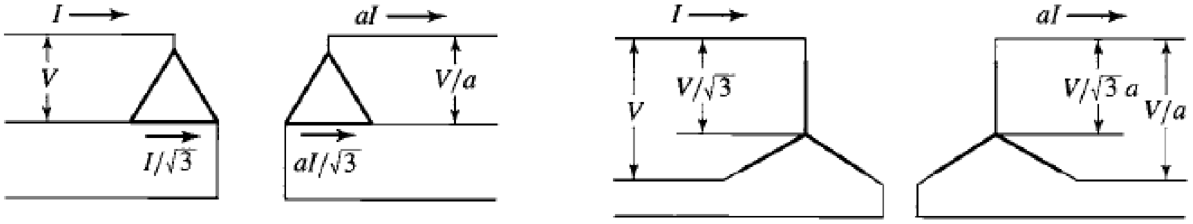
$$I = I_A + I_B = \frac{E_A\, Z_B + E_B\, Z_A}{Z_A\, Z_B + Z_L(Z_A + Z_B)} \qquad \times \frac{{}^{1}/_{Z_A\, Z_B}}{{}^{1}/_{Z_A\, Z_B}} \qquad \times Z_L$$

5-3 Transformers in Three-Phase Circuits

Three single-phase transformers can be connected to form a *three-phase transformer bank* in any of the four ways shown in Fig. 1. In all four parts of this figure, the windings at the left are the primaries, those at the right are the secondaries, and any primary winding in one transformer corresponds to the secondary winding drawn parallel to it. Also shown are the voltages and currents resulting from balanced impressed primary line-to-line voltages V and line currents I when the ratio of primary-to-secondary turns N1/N2 = ***a*** and ideal transformers are assumed. 4 Note that the rated voltages and currents at the primary and secondary of the three-phase transformer bank depends upon the connection used but that the rated kVA of the three-phase bank is three times that of the individual single-phase transformers, regardless of the connection.



(a) Y-Δ connection (b) Δ-Y connection



(c) Δ-Δ connection (d) Y-Y connection

Figure 1 Common three-phase transformer connections; the transformer windings are indicated by the heavy lines

The Y- Δ connection is commonly used in stepping down from a high voltage to a medium or low voltage. One reason is that a neutral is thereby provided for grounding on the high-voltage side, a procedure which can be shown to be desirable in many cases. Conversely, the Δ-Y connection is commonly used for stepping up to a high voltage. The Δ - Δ connection has the advantage that one transformer can be removed for repair or maintenance while the remaining two continue to function as a three-phase bank with the rating reduced to 58 percent of that of the original bank; this is known as the *open-delta,* or V, connection. The Y-Y connection is seldom used because of difficulties with exciting-current phenomena.

Instead of three single-phase transformers, a three-phase bank may consist of one *three-phase transformer* having all six windings on a common multi-legged core and contained in a single tank. Advantages of three-phase transformers over connections of three single-phase transformers are that they cost less, weigh less, require less floor space, and have somewhat higher efficiency. A photograph of the internal parts of a large three-phase transformer is shown in Fig. 2.

Circuit computations involving three-phase transformer banks under balanced conditions can be made by dealing with only one of the transformers or phases and recognizing that conditions are the same in the other two phases except for the phase displacements associated with a three-phase system. It is usually convenient to carry out the computations on a single-phase (per-phase-Y, line-to-

neutral) basis, since transformer impedances can then be added directly in series with transmission line
impedances. The impedances of transmission lines can be referred from one side of the transformer bank to the other by use of the square of the ideal line-to-line voltage

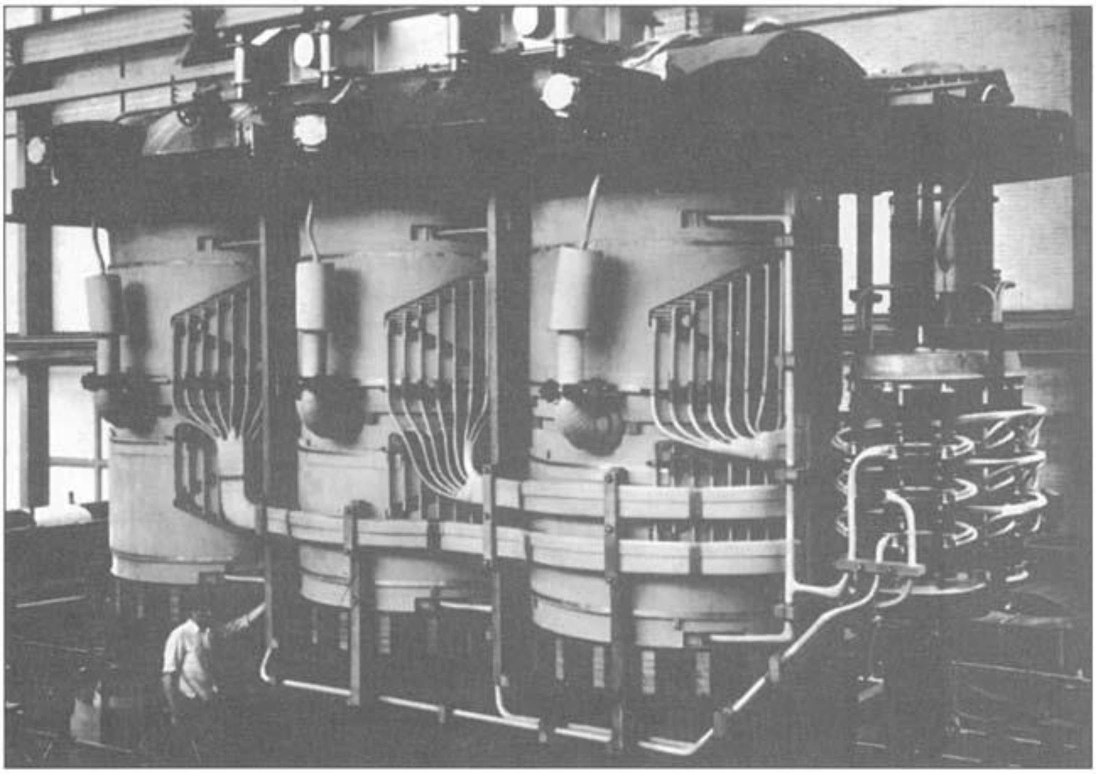


Figure 2 A 200-MVA, three-phase, 50-Hz, three-winding, 210/80/10.2-kV transformer removed from its tank. The 210-kV winding has an on-load tap changer for adjustment of the voltage.

ratio of the bank. In dealing with Y-Δor A-Y banks, all quantities can be referred to the Y-connected side. In dealing with A-A banks in series with transmission lines, it is
convenient to replace the Δ-connected impedances of the transformers by equivalent Y-connected impedances. It can be shown that a balanced A-connected circuit of ZΔ Ω/phase is equivalent to a balanced Y-connected circuit of Zy Ω/phase if

$$Z_Y = \frac{1}{3} Z_\Delta$$